Equilibrium of a Rigid Body

Question Paper 9

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Equilibrium of a Rigid Body
Sub Topic	
Booklet	Question Paper 9

Time Allowed: **57 minutes**

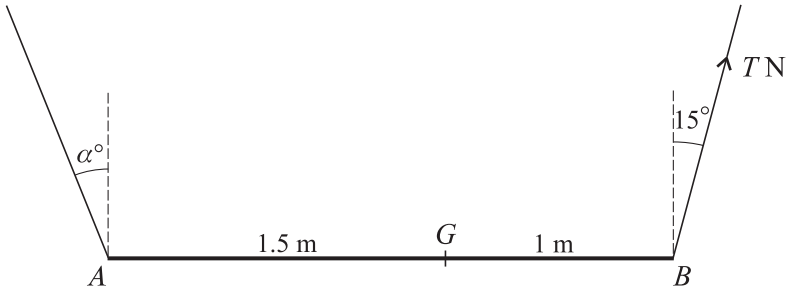
Score: **/47**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1

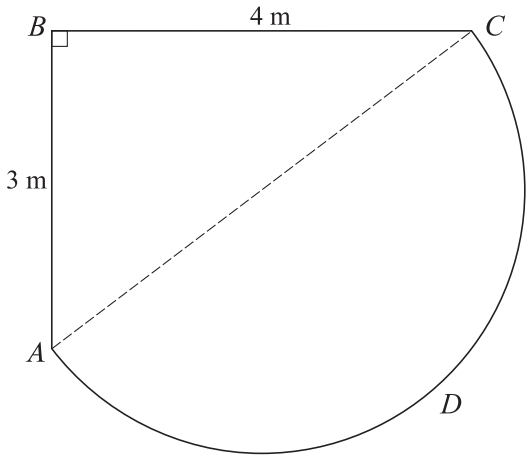


A non-uniform rod AB of length 2.5 m and mass 3 kg has its centre of mass at the point G of the rod, where $AG = 1.5$ m. The rod hangs horizontally, in equilibrium, from strings attached at A and B. The strings at A and B make angles with the vertical of $\alpha°$ and 15° respectively. The tension in the string at B is T N (see diagram). Find

(i) the value of T, [3]

(ii) the value of α. [3]

2



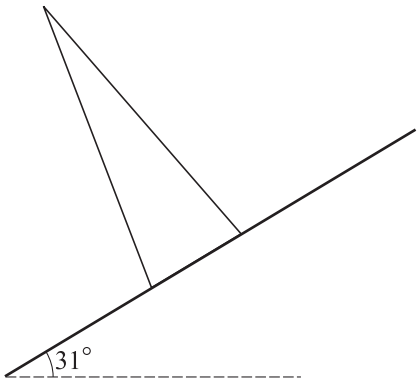
A large uniform lamina is in the shape of a right-angled triangle ABC, with hypotenuse AC, joined to a semicircle ADC with diameter AC. The sides AB and BC have lengths 3 m and 4 m respectively, as shown in the diagram.

(i) Show that the distance from AB of the centre of mass of the semicircular part ADC of the lamina is $\left(2 + \frac{2}{\pi}\right)$ m. [3]

(ii) Show that the distance from AB of the centre of mass of the complete lamina is 2.14 m, correct to 3 significant figures. [5]

3 A uniform solid cone has height 38 cm.

(i) Write down the distance of the centre of mass of the cone from its base. [1]



The cone is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted, and the cone remains in equilibrium until the angle of inclination of the plane reaches 31° (see diagram), when the cone topples.

(ii) Find the radius of the cone. [2]

(iii) Show that $\mu \geqslant 0.601$, correct to 3 significant figures, where μ is the coefficient of friction between the cone and the plane. [2]

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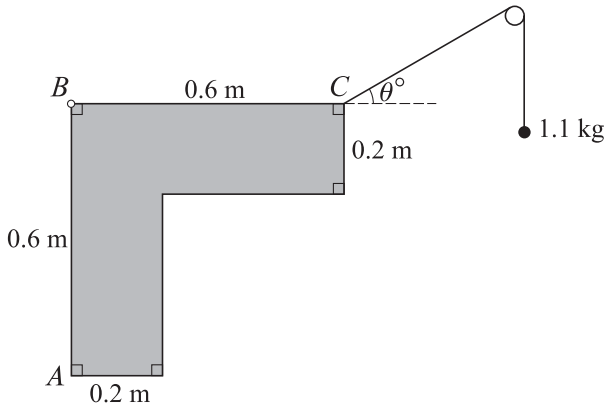

A uniform lamina of weight 15 N has dimensions as shown in the diagram.

(i) Show that the distance of the centre of mass of the lamina from AB is 0.22 m. [4]

The lamina is freely hinged at B to a fixed point. One end of a light inextensible string is attached to the lamina at C. The string passes over a fixed smooth pulley and a particle of mass 1.1 kg is attached to the other end of the string. The lamina is in equilibrium with BC horizontal. The string is taut and makes an angle of $\theta°$ with the horizontal at C, and the particle hangs freely below the pulley (see diagram).

(ii) Find the value of θ. [3]

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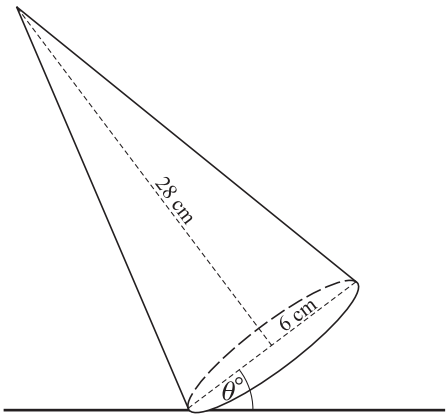

A uniform solid cone has vertical height 28 cm and base radius 6 cm. The cone is held with a point of the circumference of its base in contact with a horizontal table, and with the base making an angle of $\theta°$ with the horizontal (see diagram). When the cone is released, it moves towards the equilibrium position in which its base is in contact with the table. Show that $\theta < 40.6$, correct to 1 decimal place. [3]

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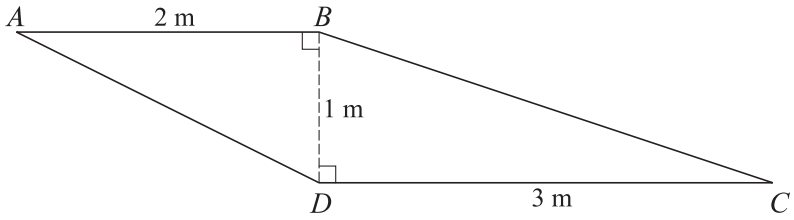

A uniform lamina $ABCD$ is in the form of a trapezium in which AB and DC are parallel and have lengths 2 m and 3 m respectively. BD is perpendicular to the parallel sides and has length 1 m (see diagram).

(i) Find the distance of the centre of mass of the lamina from BD. [3]

The lamina has weight W N and is in equilibrium, suspended by a vertical string attached to the lamina at B. The lamina rests on a vertical support at C. The lamina is in a vertical plane with AB and DC horizontal.

(ii) Find, in terms of W, the tension in the string and the magnitude of the force exerted on the lamina at C. [3]

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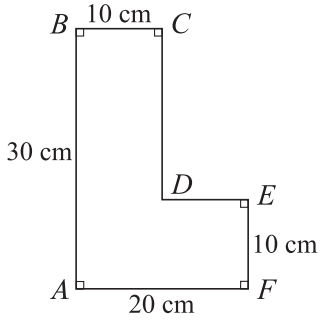

Fig. 1

ABCDEF is the L-shaped cross-section of a uniform solid. This cross-section passes through the centre of mass of the solid and has dimensions as shown in Fig. 1.

(i) Find the distance of the centre of mass of the solid from the edge *AB* of the cross-section. [3]

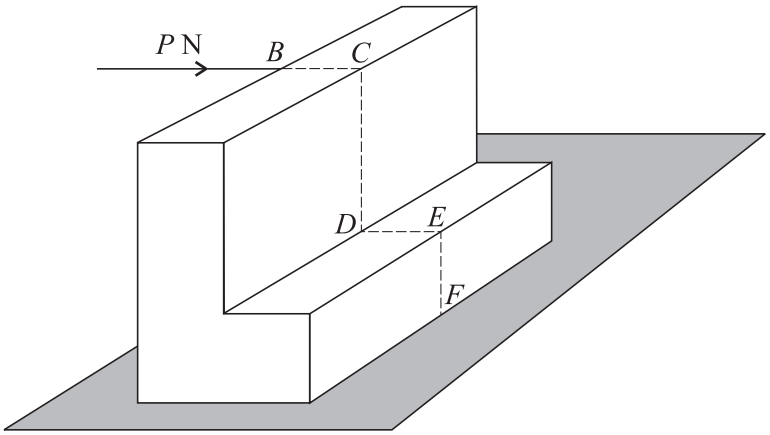

Fig. 2

The solid rests in equilibrium with the face containing the edge *AF* of the cross-section in contact with a horizontal table. The weight of the solid is *W* N. A horizontal force of magnitude *P* N is applied to the solid at the point *B*, in the direction of *BC* (see Fig. 2). The table is sufficiently rough to prevent sliding.

(ii) Find *P* in terms of *W*, given that the equilibrium of the solid is about to be broken. [3]

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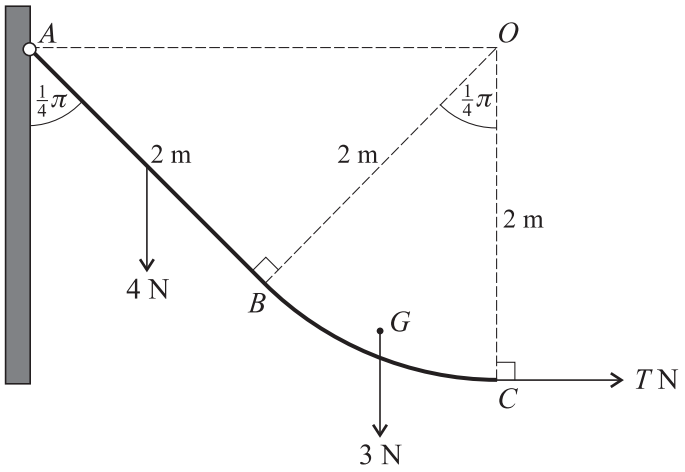

A rigid rod consists of two parts. The part BC is in the form of an arc of a circle of radius 2 m and centre O, with angle $BOC = \frac{1}{4}\pi$ radians. BC is uniform and has weight 3 N. The part AB is straight and of length 2 m; it is uniform and has weight 4 N. The part AB of the rod is a tangent to the arc BC at B. The end A of the rod is freely hinged to a fixed point of a vertical wall. The rod is held in equilibrium, with the straight part AB making an angle of $\frac{1}{4}\pi$ radians with the wall, by means of a horizontal string attached to C. The string is in the same vertical plane as the rod, and the tension in the string is T N (see diagram).

(i) Show that the centre of mass G of the part BC of the rod is at a distance of 2.083 m from the wall, correct to 4 significant figures. [4]

(ii) Find the value of T. [3]

(iii) State the magnitude of the horizontal component and the magnitude of the vertical component of the force exerted on the rod by the hinge. [1]