

Composite & Inverse Functions

Question Paper 1

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 3
Topic	Functions
Sub Topic	Composite and Inverse Functions
Booklet	Question Paper 1

Time Allowed: 62 minutes

Score: /51

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

- 1** (i) The function $f(x)$ is defined by

$$f(x) = \frac{1-x}{1+x}, x \neq -1.$$

Show that $f(f(x)) = x$.

Hence write down $f^{-1}(x)$.

[3]

- (ii) The function $g(x)$ is defined for all real x by

$$g(x) = \frac{1-x^2}{1+x^2}.$$

Prove that $g(x)$ is even. Interpret this result in terms of the graph of $y = g(x)$.

[3]

- 2 Fig. 9 shows the curve $y = f(x)$, where

$$f(x) = (e^x - 2)^2 - 1, \quad x \in \mathbb{R}.$$

The curve crosses the x -axis at O and P, and has a turning point at Q.

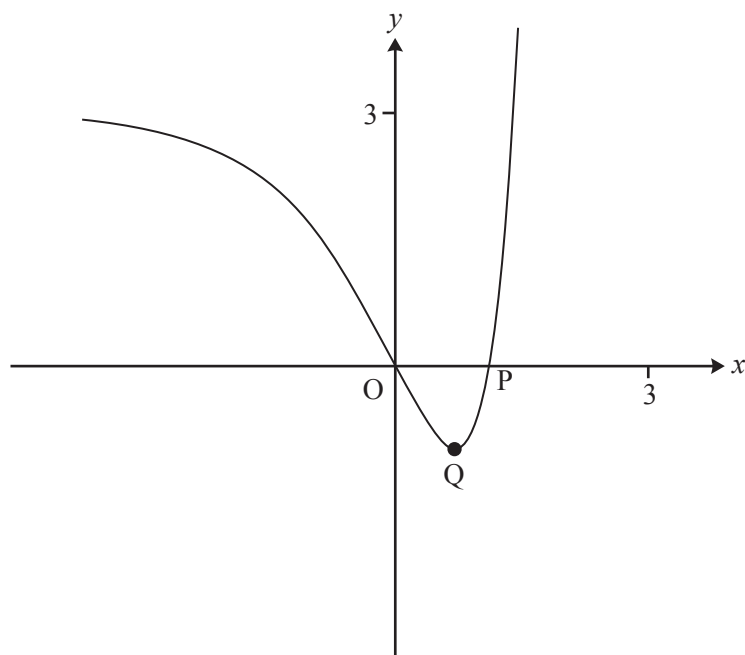


Fig. 9

- (i) Find the exact x -coordinate of P. [2]
- (ii) Show that the x -coordinate of Q is $\ln 2$ and find its y -coordinate. [4]
- (iii) Find the exact area of the region enclosed by the curve and the x -axis. [5]

The domain of $f(x)$ is now restricted to $x \geq \ln 2$.

- (iv) Find the inverse function $f^{-1}(x)$. Write down its domain and range, and sketch its graph on the copy of Fig. 9. [7]

- 3 Fig. 7 shows the curve $y = f(x)$, where $f(x) = 1 + 2 \arctan x$, $x \in \mathbb{R}$. The scales on the x - and y -axes are the same.

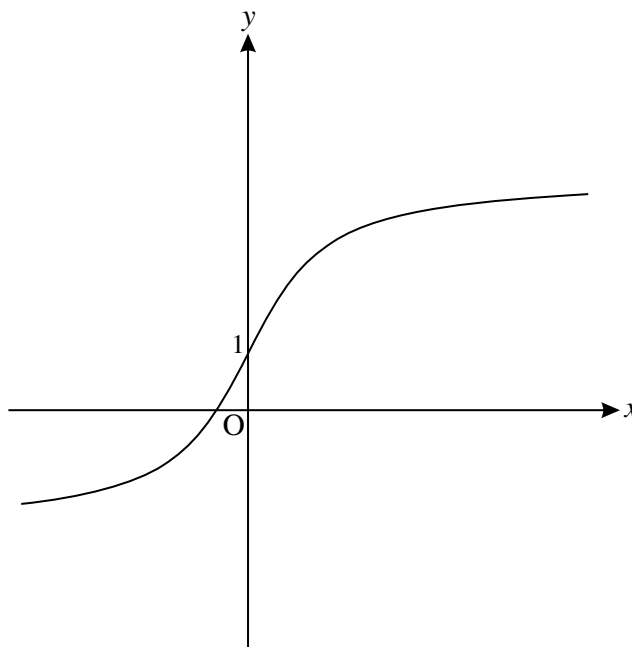


Fig. 7

- (i) Find the range of f , giving your answer in terms of π . [3]
- (ii) Find $f^{-1}(x)$, and add a sketch of the curve $y = f^{-1}(x)$ to the copy of Fig. 7. [5]
- 4 Given that $f(x) = 2 \ln x$ and $g(x) = e^x$, find the composite function $gf(x)$, expressing your answer as simply as possible. [3]

- 5** Write down the conditions for $f(x)$ to be an odd function and for $g(x)$ to be an even function.

Hence prove that, if $f(x)$ is odd and $g(x)$ is even, then the composite function $gf(x)$ is even. **[4]**

- 6** The function $f(x)$ is defined by

$$f(x) = 1 + 2 \sin 3x, \quad -\frac{\pi}{6} \leq x \leq \frac{\pi}{6}.$$

You are given that this function has an inverse, $f^{-1}(x)$.

Find $f^{-1}(x)$ and its domain. **[6]**

- 7** Given that $f(x) = \frac{1}{2} \ln(x - 1)$ and $g(x) = 1 + e^{2x}$, show that $g(x)$ is the inverse of $f(x)$. **[3]**

- 8** Sketch the curve $y = 2 \arccos x$ for $-1 \leq x \leq 1$. **[3]**