

Composite & Inverse Functions

Question Paper 2

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 3
Topic	Functions
Sub Topic	Composite and Inverse Functions
Booklet	Question Paper 2

Time Allowed: 64 minutes

Score: /53

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

- 1 Given that $f(x) = \frac{x+1}{x-1}$, show that $ff(x) = x$.

Hence write down the inverse function $f^{-1}(x)$. What can you deduce about the symmetry of the curve $y = f(x)$? [5]

- 2 The functions $f(x)$ and $g(x)$ are defined for all real numbers x by

$$f(x) = x^2, \quad g(x) = x - 2.$$

(i) Find the composite functions $fg(x)$ and $gf(x)$. [3]

(ii) Sketch the curves $y = f(x)$, $y = fg(x)$ and $y = gf(x)$, indicating clearly which is which. [2]

- 3 Given that $f(x) = 1 - x$ and $g(x) = |x|$, write down the composite function $gf(x)$.

On separate diagrams, sketch the graphs of $y = f(x)$ and $y = gf(x)$. [3]

- 4 The function $f(x)$ is defined by $f(x) = 1 + 2\sin x$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

(i) Show that $f^{-1}(x) = \arcsin\left(\frac{x-1}{2}\right)$ and state the domain of this function. [4]

Fig. 6 shows a sketch of the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

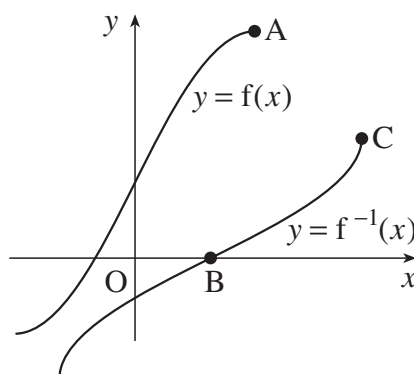


Fig. 6

(ii) Write down the coordinates of the points A, B and C. [3]

- 5 Given that $\arcsin x = \frac{1}{6}\pi$, find x . Find $\arccos x$ in terms of π . [3]

- 6 The functions $f(x)$ and $g(x)$ are defined for the domain $x > 0$ as follows:

$$f(x) = \ln x, \quad g(x) = x^3.$$

Express the composite function $fg(x)$ in terms of $\ln x$.

State the transformation which maps the curve $y = f(x)$ onto the curve $y = fg(x)$. [3]

- 7 Fig. 9 shows the line $y = x$ and the curve $y = f(x)$, where $f(x) = \frac{1}{2}(e^x - 1)$. The line and the curve intersect at the origin and at the point $P(a, a)$.

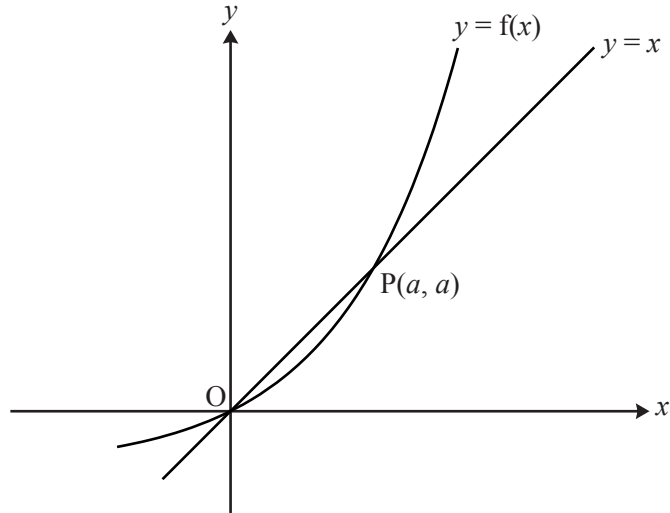


Fig. 9

- (i) Show that $e^a = 1 + 2a$. [1]
- (ii) Show that the area of the region enclosed by the curve, the x -axis and the line $x = a$ is $\frac{1}{2}a$. Hence find, in terms of a , the area enclosed by the curve and the line $y = x$. [6]
- (iii) Show that the inverse function of $f(x)$ is $g(x)$, where $g(x) = \ln(1 + 2x)$. Add a sketch of $y = g(x)$ to the copy of Fig. 9. [5]
- (iv) Find the derivatives of $f(x)$ and $g(x)$. Hence verify that $g'(a) = \frac{1}{f'(a)}$.
Give a geometrical interpretation of this result. [7]

- 8 The function $f(x)$ is defined by $f(x) = 1 - 2 \sin x$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$. Fig. 3 shows the curve $y = f(x)$.

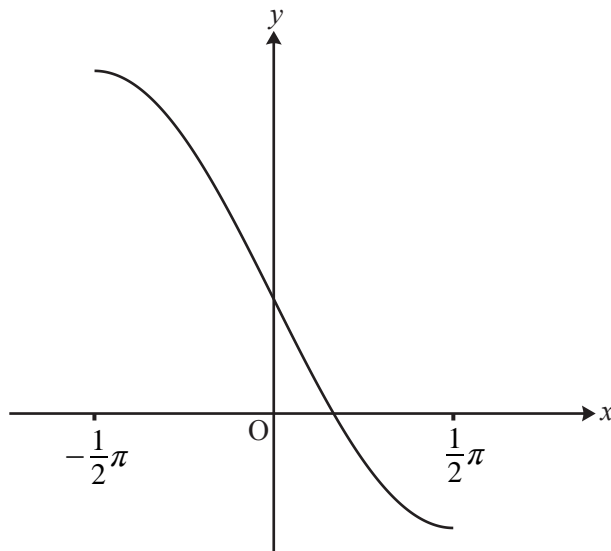


Fig. 3

- (i) Write down the range of the function $f(x)$. [2]
- (ii) Find the inverse function $f^{-1}(x)$. [3]
- (iii) Find $f'(0)$. Hence write down the gradient of $y = f^{-1}(x)$ at the point $(1, 0)$. [3]