Linear Motion under a Variable Force

Question Paper 1

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Linear Motion under a Variable Force
Sub Topic	
Booklet	Question Paper 1

Time Allowed: **63 minutes**

Score: **/52**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A particle P of mass 0.1 kg moves with decreasing speed in a straight line on a smooth horizontal surface. A horizontal resisting force of magnitude $0.2e^{-x}$ N acts on P, where x m is the displacement of P from a fixed point O on the line. The velocity of P is v m s^{-1} when its displacement from O is x m.

(i) Show that

$$v\frac{dv}{dx} = ke^{-x},$$

where k is a constant to be found. [2]

P passes through O with velocity 2.2 m s^{-1}.

(ii) Calculate the value of x at the instant when the velocity of P is 2 m s^{-1}. [4]

(iii) Show that the speed of P does not fall below 0.917 m s^{-1}, correct to 3 significant figures. [2]

2 A force of magnitude $0.4t$ N, applied at an angle of 30° above the horizontal, acts on a particle P, where t s is the time since the force starts to act. P is at rest on rough horizontal ground when $t = 0$. The mass of P is 0.2 kg and the coefficient of friction between P and the ground is μ.

(i) Given that P is about to slip when $t = 2$, find μ and the value of t for the instant when P loses contact with the ground. [5]

(ii) While P is moving on the ground, it has velocity v m s^{-1} at time t s. Show that

$$\frac{dv}{dt} = 2.165t - 4.330,$$

where the coefficients are correct to 4 significant figures. [3]

(iii) Calculate the speed of P when it loses contact with the ground. [4]

3 A cyclist and her bicycle have a total mass of 60 kg. The cyclist rides in a horizontal straight line, and exerts a constant force in the direction of motion of 150 N. The motion is opposed by a resistance of magnitude $12v$ N, where v m s^{-1} is the cyclist's speed at time t s after passing through a fixed point A.

(i) Show that $5\frac{dv}{dt} = 12.5 - v$. [2]

(ii) Given that the cyclist passes through A with speed 11.5 m s^{-1}, solve this differential equation to show that $v = 12.5 - e^{-0.2t}$. [4]

(iii) Express the displacement of the cyclist from A in terms of t. [3]

4 O, A and B are three points in a straight line on a smooth horizontal surface. A particle P of mass 0.6 kg moves along the line. At time t s the particle has displacement x m from O and speed v m s^{-1}. The only horizontal force acting on P has magnitude $0.4v^{\frac{1}{2}}$ N and acts in the direction OA. Initially the particle is at A, where $x = 1$ and $v = 1$.

(i) Show that $3v^{\frac{1}{2}}\dfrac{dv}{dx} = 2$. [2]

(ii) Express v in terms of x. [4]

(iii) Given that $AB = 7$ m, find the value of t when P passes through B. [3]

5 A small ball of mass m kg is projected vertically upwards with speed 14 m s^{-1}. The ball has velocity v m s^{-1} upwards when it is x m above the point of projection. A resisting force of magnitude $0.02mv$ N acts on the ball during its upward motion.

(i) Show that, while the ball is moving upwards, $\left(\dfrac{500}{v+500} - 1\right)\dfrac{dv}{dx} = 0.02$. [3]

(ii) Find the greatest height of the ball above its point of projection. [3]

6 A small block B of mass 0.2 kg is placed at a fixed point O on a smooth horizontal surface. A horizontal force of magnitude 0.42 N is applied to B. At time t s after the force is first applied, the velocity of B away from O is v m s^{-1}.

(i) Find the value of v when $t = 1$. [2]

For $t > 1$ an additional force, of magnitude $0.32t$ N and directed towards O, is applied to B. The force of magnitude 0.42 N continues to act as before.

(ii) Find the value of v when $t = 2$. [3]

For $t > 2$ a third force, of magnitude $0.06t^2$ N and directed away from O, is applied to B. The other two forces continue to act as before.

(iii) Show that the velocity of B is the same when $t = 2$ and when $t = 3$. [3]