Linear Motion under a Variable Force

Question Paper 4

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Linear Motion under a Variable Force
Sub Topic	
Booklet	Question Paper 4

Time Allowed: **62 minutes**

Score: **/51**

Percentage: **/100**

Grade Boundaries:

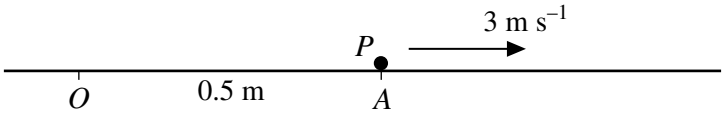
A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A stone is thrown with speed $15\,\text{m s}^{-1}$ horizontally from the top of a vertical cliff 20 m above the sea. Calculate

(i) the distance from the foot of the cliff to the point where the stone enters the sea, [3]

(ii) the speed of the stone when it enters the sea. [3]

2



O and A are fixed points on a horizontal surface, with $OA = 0.5\,\text{m}$. A particle P of mass 0.2 kg is projected horizontally with speed $3\,\text{m s}^{-1}$ from A in the direction OA and moves in a straight line (see diagram). At time t s after projection, the velocity of P is $v\,\text{m s}^{-1}$ and its displacement from O is x m. The coefficient of friction between the surface and P is 0.5, and a force of magnitude $\frac{0.4}{x^2}$ N acts on P in the direction PO.

(i) Show that, while the particle is in motion, $v\frac{\mathrm{d}v}{\mathrm{d}x} = -\left(5 + \frac{2}{x^2}\right)$. [2]

(ii) Calculate the distance travelled by P before it comes to rest, and show that P does not subsequently move. [7]

3 A cyclist and his bicycle have a total mass of 81 kg. The cyclist starts from rest and rides in a straight line. The cyclist exerts a constant force of 135 N and the motion is opposed by a resistance of magnitude $9v$ N, where $v\,\text{m s}^{-1}$ is the cyclist's speed at time t s after starting.

(i) Show that $\frac{9}{15 - v}\frac{\mathrm{d}v}{\mathrm{d}t} = 1$. [2]

(ii) Solve this differential equation to show that $v = 15\left(1 - \mathrm{e}^{-\frac{1}{9}t}\right)$. [4]

(iii) Find the distance travelled by the cyclist in the first 9 s of the motion. [4]

4 A particle P of mass 0.25 kg moves in a straight line on a smooth horizontal surface. P starts at the point O with speed 10 m s^{-1} and moves towards a fixed point A on the line. At time t s the displacement of P from O is x m and the velocity of P is v m s^{-1}. A resistive force of magnitude $(5 - x)$ N acts on P in the direction towards O.

(i) Form a differential equation in v and x. By solving this differential equation, show that $v = 10 - 2x$. [6]

(ii) Find x in terms of t, and hence show that the particle is always less than 5 m from O. [5]

5 A particle P of mass 0.5 kg moves in a straight line on a smooth horizontal surface. At time t s, the displacement of P from a fixed point on the line is x m and the velocity of P is v m s^{-1}. It is given that when $t = 0$, $x = 0$ and $v = 9$. The motion of P is opposed by a force of magnitude $3\sqrt{v}$ N.

(i) By solving an appropriate differential equation, show that $v = (27 - 9x)^{\frac{2}{3}}$. [5]

(ii) Calculate the value of x when $t = 0.5$. [4]

6 A particle P starts from a fixed point O and moves in a straight line. When the displacement of P from O is x m, its velocity is v m s^{-1} and its acceleration is $\dfrac{1}{x+2}$ m s^{-2}.

(i) Given that $v = 2$ when $x = 0$, use integration to show that $v^2 = 2\ln\left(\frac{1}{2}x + 1\right) + 4$. [4]

(ii) Find the value of v when the acceleration of P is $\frac{1}{4}$ m s^{-2}. [2]