Linear Motion under a Variable Force

Question Paper 5

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Linear Motion under a Variable Force
Sub Topic	
Booklet	Question Paper 5

Time Allowed: **62 minutes**

Score: **/51**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A particle P of mass 0.5 kg moves along the x-axis on a horizontal surface. When the displacement of P from the origin O is x m the velocity of P is v m s^{-1} in the positive x-direction. Two horizontal forces act on P; one force has magnitude $(1 + 0.3x^2)$ N and acts in the positive x-direction, and the other force has magnitude $8e^{-x}$ N and acts in the negative x-direction.

(i) Show that $v\dfrac{dv}{dx} = 2 + 0.6x^2 - 16e^{-x}$. [2]

(ii) The velocity of P as it passes through O is 6 m s^{-1}. Find the velocity of P when $x = 3$. [5]

2 A particle P of mass 0.5 kg moves on a horizontal surface along the straight line OA, in the direction from O to A. The coefficient of friction between P and the surface is 0.08. Air resistance of magnitude $0.2v$ N opposes the motion, where v m s^{-1} is the speed of P at time t s. The particle passes through O with speed 4 m s^{-1} when $t = 0$.

(i) Show that $2.5\dfrac{dv}{dt} = -(v + 2)$ and hence find the value of t when $v = 0$. [7]

(ii) Show that $\dfrac{dx}{dt} = 6e^{-0.4t} - 2$, where x m is the displacement of P from O at time t s, and hence find the distance OP when $v = 0$. [5]

3 A particle of mass 0.4 kg is released from rest and falls vertically. A resisting force of magnitude $0.08v$ N acts upwards on the particle during its descent, where v m s^{-1} is the velocity of the particle at time t s after its release.

(i) Show that the acceleration of the particle is $(10 - 0.2v)$ m s^{-2}. [2]

(ii) Find the velocity of the particle when $t = 15$. [5]

4 A particle starts from rest at O and travels in a straight line. Its acceleration is $(3 - 2x)\ \text{m s}^{-2}$, where x m is the displacement of the particle from O.

(i) Find the value of x for which the velocity of the particle reaches its maximum value. [1]

(ii) Find this maximum velocity. [4]

5 A cyclist starts from rest at a point O and travels along a straight path. At time t s after starting, the displacement of the cyclist from O is x m, and the acceleration of the cyclist is $a\ \text{m s}^{-2}$, where $a = 0.6x^{0.2}$.

(i) Find an expression for the velocity $v\ \text{m s}^{-1}$ of the cyclist in terms of x. [4]

(ii) Show that $t = 2.5x^{0.4}$. [3]

(iii) Find the distance travelled by the cyclist in the first 10 s of the journey. [2]

6 A particle of mass 0.25 kg moves in a straight line on a smooth horizontal surface. A variable resisting force acts on the particle. At time t s the displacement of the particle from a point on the line is x m, and its velocity is $(8 - 2x)\ \text{m s}^{-1}$. It is given that $x = 0$ when $t = 0$.

(i) Find the acceleration of the particle in terms of x, and hence find the magnitude of the resisting force when $x = 1$. [3]

(ii) Find an expression for x in terms of t. [6]

(iii) Show that the particle is always less than 4 m from its initial position. [2]