Linear Motion under a Variable Force

Question Paper 6

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Linear Motion under a Variable Force
Sub Topic	
Booklet	Question Paper 6

Time Allowed: **74 minutes**

Score: **/61**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The acceleration of a particle moving in a straight line is $(x - 2.4)\,\text{m s}^{-2}$ when its displacement from a fixed point O of the line is x m. The velocity of the particle is $v\,\text{m s}^{-1}$, and it is given that $v = 2.5$ when $x = 0$. Find

(i) an expression for v in terms of x, [5]

(ii) the minimum value of v. [2]

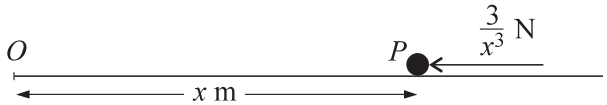
2 A car of mass 1000 kg is moving on a straight horizontal road. The driving force of the car is $\frac{28\,000}{v}$ N and the resistance to motion is $4v$ N, where $v\,\text{m s}^{-1}$ is the speed of the car t seconds after it passes a fixed point on the road.

(i) Show that $\frac{\text{d}v}{\text{d}t} = \frac{7000 - v^2}{250v}$. [2]

The car passes points A and B with speeds $10\,\text{m s}^{-1}$ and $40\,\text{m s}^{-1}$ respectively.

(ii) Find the time taken for the car to travel from A to B. [4]

3



A particle P of mass 0.6 kg moves in a straight line on a smooth horizontal surface. A force of magnitude $\frac{3}{x^3}$ newtons acts on the particle in the direction from P to O, where O is a fixed point of the surface and x m is the distance OP (see diagram). The particle P is released from rest at the point where $x = 10$. Find the speed of P when $x = 2.5$. [7]

4 One end of a light inextensible string of length 0.15 m is attached to a fixed point which is above a smooth horizontal surface. A particle of mass 0.5 kg is attached to the other end of the string. The particle moves with constant speed v m s^{-1} in a horizontal circle, with the string taut and making an angle of $\theta°$ with the downward vertical.

(i) Given that $\theta = 60$ and that the particle is not in contact with the surface, find v. [5]

(ii) Given instead that $\theta = 45$ and $v = 0.9$, and that the particle is in contact with the surface, find

(a) the tension in the string, [2]

(b) the force exerted by the surface on the particle. [3]

5 A cyclist and his machine have a total mass of 80 kg. The cyclist starts from rest and rides from the bottom to the top of a straight slope inclined at an angle θ to the horizontal, where $\sin\theta = 0.1$. The cyclist exerts a constant force of magnitude 120 N. There is a resisting force of magnitude $8v$ N acting on the cyclist, where v m s^{-1} is the cyclist's speed at time t s after the start.

(i) Show that $\left(\frac{1}{5-v}\right)\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{10}$. [3]

(ii) Solve this differential equation and hence show that $v = 5\left(1 - \mathrm{e}^{-\frac{1}{10}t}\right)$. [5]

(iii) Given that the cyclist takes 20 s to reach the top of the slope, find the length of the slope. [4]

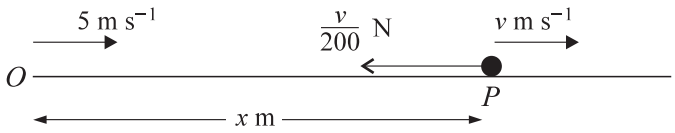
6 A particle is projected with speed 60 m s^{-1} from a point on horizontal ground. The angle of projection is $\alpha°$ above the horizontal. The particle reaches the ground again after 10 s.

(i) Find the value of α. [3]

(ii) Find the greatest height reached by the particle. [2]

(iii) At time T s after the instant of projection the direction of motion of the particle is at an angle of 45° above the horizontal. Find the value of T. [4]

7



A particle P of mass $\frac{1}{10}$ kg travels in a straight line on a smooth horizontal surface. It passes through the fixed point O with velocity $5\,\text{m}\,\text{s}^{-1}$ at time $t = 0$. After t seconds its displacement from O is x m and its velocity is $v\,\text{m}\,\text{s}^{-1}$. P is subject to a single force of magnitude $\frac{v}{200}$ N which acts in a direction opposite to the motion (see diagram).

(i) Find an expression for v in terms of x. [4]

(ii) Find an expression for x in terms of t. [5]

(iii) Show that the value of x is less than 100 for all values of t. [1]