Differentiation – Inverse Functions

Question Paper 3

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 3
Topic	Calculus
Sub Topic	Differentiation – Inverse Functions
Booklet	Question Paper 3

Time Allowed: **54 minutes**

Score: **/45**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 Fig. 8 shows part of the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (\mathrm{e}^x - 1)^2 \quad \text{for } x \geqslant 0.$$

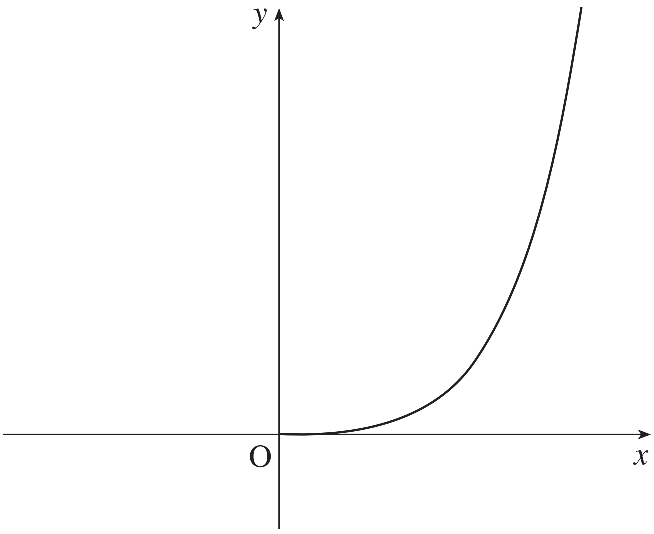

Fig. 8

(i) Find $\mathrm{f}'(x)$, and hence calculate the gradient of the curve $y = \mathrm{f}(x)$ at the origin and at the point $(\ln 2, 1)$. [5]

The function $\mathrm{g}(x)$ is defined by $\sqrt{}$ for $x \geqslant 0$.

(ii) Show that $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are inverse functions. Hence sketch the graph of $y = \mathrm{g}(x)$.

Write down the gradient of the curve $y = \mathrm{g}(x)$ at the point $(1, \ln 2)$. [5]

(iii) Show that $\int (\mathrm{e}^x - 1)^2 \, \mathrm{d}x = \frac{1}{2}\mathrm{e}^{2x} - 2\mathrm{e}^x + x + c$.

Hence evaluate $\int_0^{\ln 2} (\mathrm{e}^x - 1)^2 \, \mathrm{d}x$, giving your answer in an exact form. [5]

(iv) Using your answer to part **(iii)**, calculate the area of the region enclosed by the curve $y = \mathrm{g}(x)$, the x-axis and the line $x = 1$. [3]

2 Fig. 6 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{1}{2}\arctan x$.

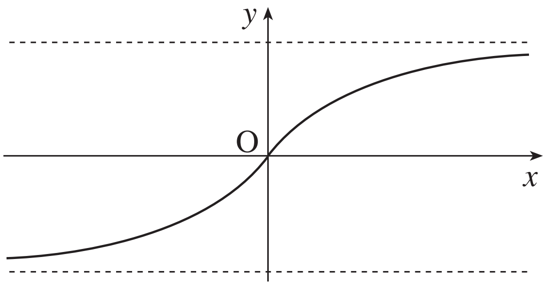

Fig. 6

(i) Find the range of the function $\mathrm{f}(x)$, giving your answer in terms of π. [2]

(ii) Find the inverse function $\mathrm{f}^{-1}(x)$. Find the gradient of the curve $y = \mathrm{f}^{-1}(x)$ at the origin. [5]

(iii) Hence write down the gradient of $y = \frac{1}{2}\arctan x$ at the origin. [1]

3 The function $f(x) = \ln(1 + x^2)$ has domain $-3 \leqslant x \leqslant 3$.

Fig. 9 shows the graph of $y = f(x)$.

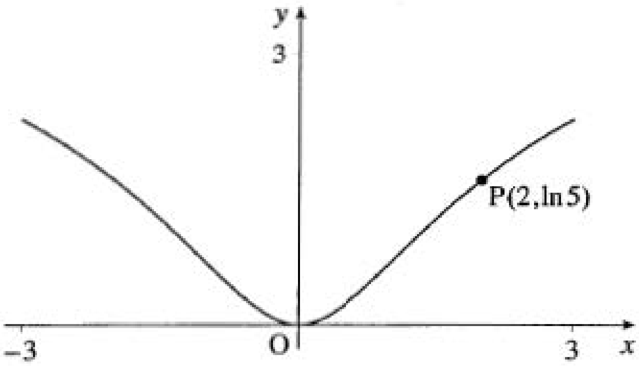

Fig. 9

(i) Show algebraically that the function is even. State how this property relates to the shape of the curve. [3]

(ii) Find the gradient of the curve at the point $P(2, \ln 5)$. [4]

(iii) Explain why the function does not have an inverse for the domain $-3 \leqslant x \leqslant 3$. [1]

The domain of $f(x)$ is now restricted to $0 \leqslant x \leqslant 3$. The inverse of $f(x)$ is the function $g(x)$.

(iv) Sketch the curves $y = f(x)$ and $y = g(x)$ on the same axes.

State the domain of the function $g(x)$.

Show that $g(x) = \sqrt{e^x - 1}$. [6]

(v) Differentiate $g(x)$. Hence verify that $g'(\ln 5) = 1\frac{1}{4}$. Explain the connection between this result and your answer to part **(ii)**. [5]