

Implicit Differentiation

Question Paper 2

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 3
Topic	Calculus
Sub Topic	Implicit Differentiation
Booklet	Question Paper 2

Time Allowed: 57 minutes

Score: /47

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 (i) Differentiate $\sqrt{1 + 3x^2}$. [3]

(ii) Hence show that the derivative of $x\sqrt{1 + 3x^2}$ is $\frac{1 + 6x^2}{\sqrt{1 + 3x^2}}$. [4]

2 Given that $y^3 = xy - x^2$, show that $\frac{dy}{dx} = \frac{y - 2x}{3y^2 - x}$.

Hence show that the curve $y^3 = xy - x^2$ has a stationary point when $x = \frac{1}{8}$. [7]

- 3 Fig. 8 shows the curve $y = x^2 - \frac{1}{8} \ln x$. P is the point on this curve with x -coordinate 1, and R is the point $(0, -\frac{7}{8})$.

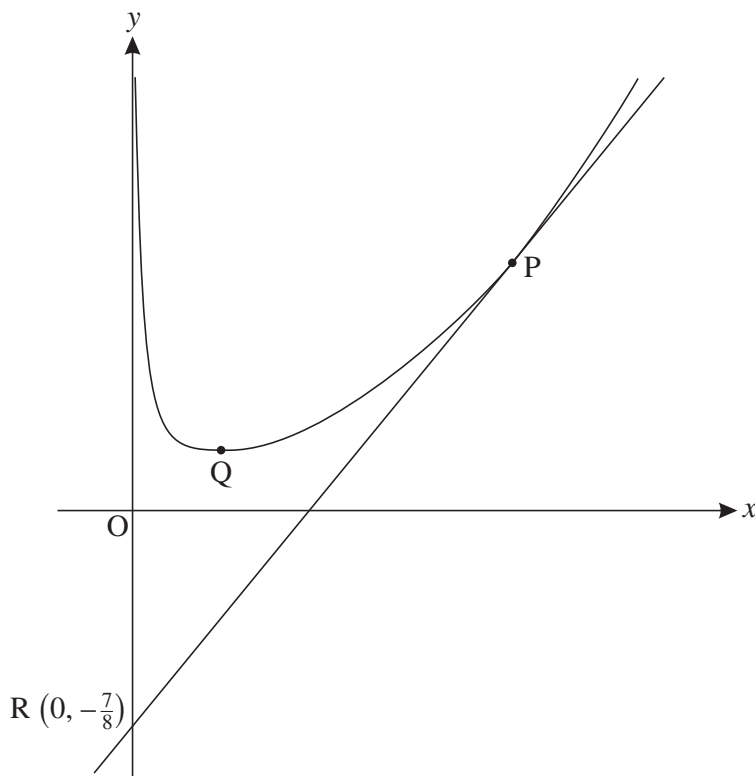


Fig. 8

- (i) Find the gradient of PR. [3]
- (ii) Find $\frac{dy}{dx}$. Hence show that PR is a tangent to the curve. [3]
- (iii) Find the exact coordinates of the turning point Q. [5]
- (iv) Differentiate $x \ln x - x$.

Hence, or otherwise, show that the area of the region enclosed by the curve $y = x^2 - \frac{1}{8} \ln x$, the x -axis and the lines $x = 1$ and $x = 2$ is $\frac{59}{24} - \frac{1}{4} \ln 2$. [7]

4 The equation of a curve is given by $e^{2y} = 1 + \sin x$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y . [3]

(ii) Find an expression for y in terms of x , and differentiate it to verify the result in part (i). [4]

5 Fig. 6 shows the curve $e^{2y} = x^2 + y$.

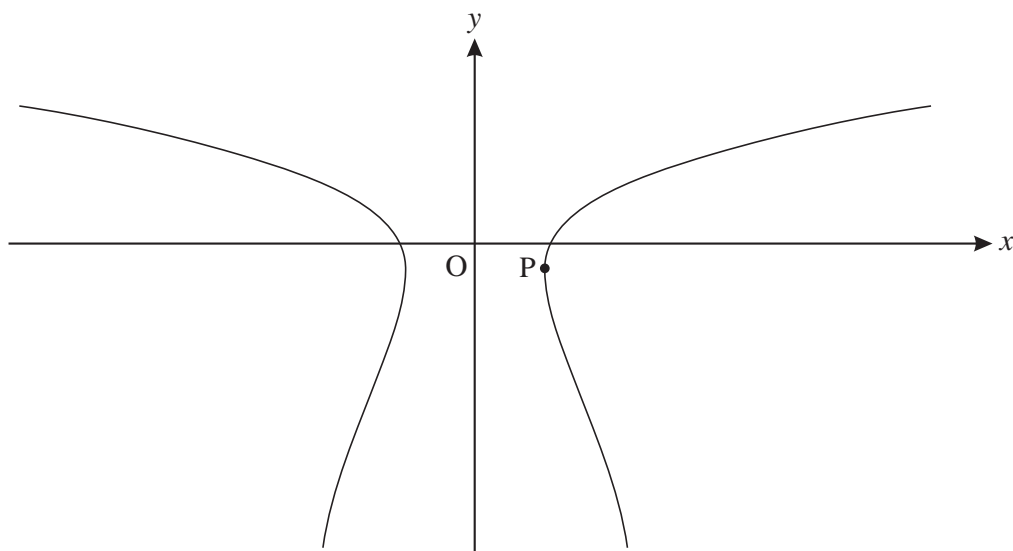


Fig. 6

(i) Show that $\frac{dy}{dx} = \frac{2x}{2e^{2y} - 1}$. [4]

(ii) Hence find to 3 significant figures the coordinates of the point P, shown in Fig. 6, where the curve has infinite gradient. [4]