

Differential Equations

Question Paper 1

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 4
Topic	Calculus
Sub Topic	Differential Equations
Booklet	Question Paper 1

Time Allowed: 83 minutes

Score: /692

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 A drug is administered by an intravenous drip. The concentration, x , of the drug in the blood is measured as a fraction of its maximum level. The drug concentration after t hours is modelled by the differential equation

$$\frac{dx}{dt} = k(1 + x - 2x^2),$$

where $0 \leq x < 1$, and k is a positive constant. Initially, $x = 0$.

(i) Express $\frac{1}{(1+2x)(1-x)}$ in partial fractions. [3]

(ii) Hence solve the differential equation to show that $\frac{1+2x}{1-x} = e^{3kt}$. [7]

- (iii) After 1 hour the drug concentration reaches 75% of its maximum value and so $x = 0.75$.

Find the value of k , and the time taken for the drug concentration to reach 90% of its maximum value. [3]

(iv) Rearrange the equation in part (ii) to show that $x = \frac{1 - e^{-3kt}}{1 + 2e^{-3kt}}$.

Verify that in the long term the drug concentration approaches its maximum value. [5]

- 2 A curve has parametric equations $x = e^{3t}$, $y = te^{2t}$.

(i) Find $\frac{dy}{dx}$ in terms of t . Hence find the exact gradient of the curve at the point with parameter $t = 1$. [4]

- (ii) Find the cartesian equation of the curve in the form $y = ax^b \ln x$, where a and b are constants to be determined. [3]

- 3 Fig. 8.1 shows an upright cylindrical barrel containing water. The water is leaking out of a hole in the side of the barrel.

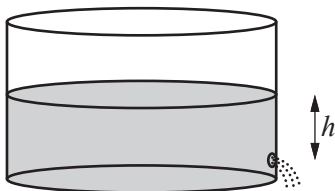


Fig. 8.1

The height of the water surface above the hole t seconds after opening the hole is h metres, where

$$\frac{dh}{dt} = -A\sqrt{h}$$

and where A is a positive constant. Initially the water surface is 1 metre above the hole.

- (i) Verify that the solution to this differential equation is

$$h = \left(1 - \frac{1}{2}At\right)^2. \quad [3]$$

The water stops leaking when $h = 0$. This occurs after 20 seconds.

- (ii) Find the value of A , and the time when the height of the water surface above the hole is 0.5 m. [4]

Fig. 8.2 shows a similar situation with a different barrel; h is in metres.

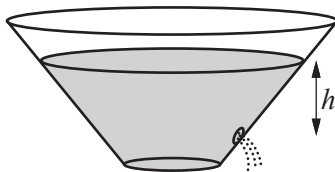


Fig. 8.2

For this barrel,

$$\frac{dh}{dt} = -B\frac{\sqrt{h}}{(1+h)^2},$$

where B is a positive constant. When $t = 0$, $h = 1$.

- (iii) Solve this differential equation, and hence show that

$$h^{\frac{1}{2}}(30 + 20h + 6h^2) = 56 - 15Bt. \quad [7]$$

- (iv) Given that $h = 0$ when $t = 20$, find B .

Find also the time when the height of the water surface above the hole is 0.5 m. [4]

- 4 The motion of a particle is modelled by the differential equation

$$v \frac{dv}{dx} + 4x = 0,$$

where x is its displacement from a fixed point, and v is its velocity.

Initially $x = 1$ and $v = 4$.

- (i) Solve the differential equation to show that $v^2 = 20 - 4x^2$. [4]

Now consider motion for which $x = \cos 2t + 2 \sin 2t$, where x is the displacement from a fixed point at time t .

- (ii) Verify that, when $t = 0$, $x = 1$. Use the fact that $v = \frac{dx}{dt}$ to verify that when $t = 0$, $v = 4$. [4]

- (iii) Express x in the form $R \cos(2t - \alpha)$, where R and α are constants to be determined, and obtain the corresponding expression for v . Hence or otherwise verify that, for this motion too, $v^2 = 20 - 4x^2$. [7]

- (iv) Use your answers to part (iii) to find the maximum value of x , and the earliest time at which x reaches this maximum value. [3]

- 5 The total value of the sales made by a new company in the first t years of its existence is denoted by $\text{£}V$. A model is proposed in which the rate of increase of V is proportional to the square root of V . The constant of proportionality is k .

- (i) Express the model as a differential equation.

Verify by differentiation that $V = (\frac{1}{2}kt + c)^2$, where c is an arbitrary constant, satisfies this differential equation. [4]

- (ii) The value of the company's sales in its first year is $\text{£}10\,000$, and the total value of the sales in the first two years is $\text{£}40\,000$. Find V in terms of t . [4]