

# Lines & Planes

## Question Paper 2

<b>Level</b>	A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	OCR - MEI
<b>Module</b>	Core 4
<b>Topic</b>	Vectors
<b>Sub Topic</b>	Lines & Planes
<b>Booklet</b>	Question Paper 2

**Time Allowed:** 84 minutes

**Score:** /70

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 With respect to cartesian coordinates  $Oxyz$ , a laser beam  $ABC$  is fired from the point  $A(1, 2, 4)$ , and is reflected at point  $B$  off the plane with equation  $x + 2y - 3z = 0$ , as shown in Fig. 8.  $A'$  is the point  $(2, 4, 1)$ , and  $M$  is the midpoint of  $AA'$ .

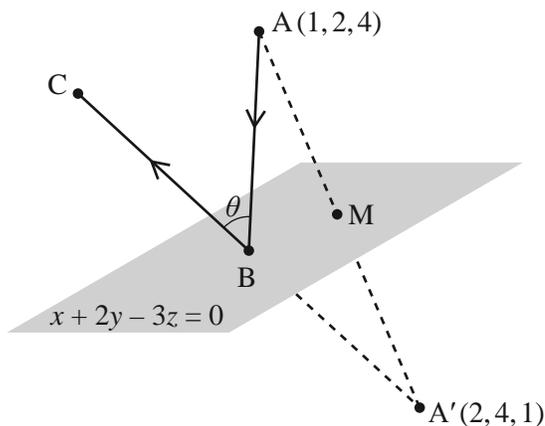


Fig. 8

- (i) Show that  $AA'$  is perpendicular to the plane  $x + 2y - 3z = 0$ , and that  $M$  lies in the plane. [4]

The vector equation of the line  $AB$  is  $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ .

- (ii) Find the coordinates of  $B$ , and a vector equation of the line  $A'B$ . [6]
- (iii) Given that  $A'BC$  is a straight line, find the angle  $\theta$ . [4]
- (iv) Find the coordinates of the point where  $BC$  crosses the  $Oxz$  plane (the plane containing the  $x$ - and  $z$ -axes) [3]

- 2 A piece of cloth ABDC is attached to the tops of vertical poles AE, BF, DG and CH, where E, F, G and H are at ground level (see Fig. 7). Coordinates are as shown, with lengths in metres. The length of pole DG is  $k$  metres.

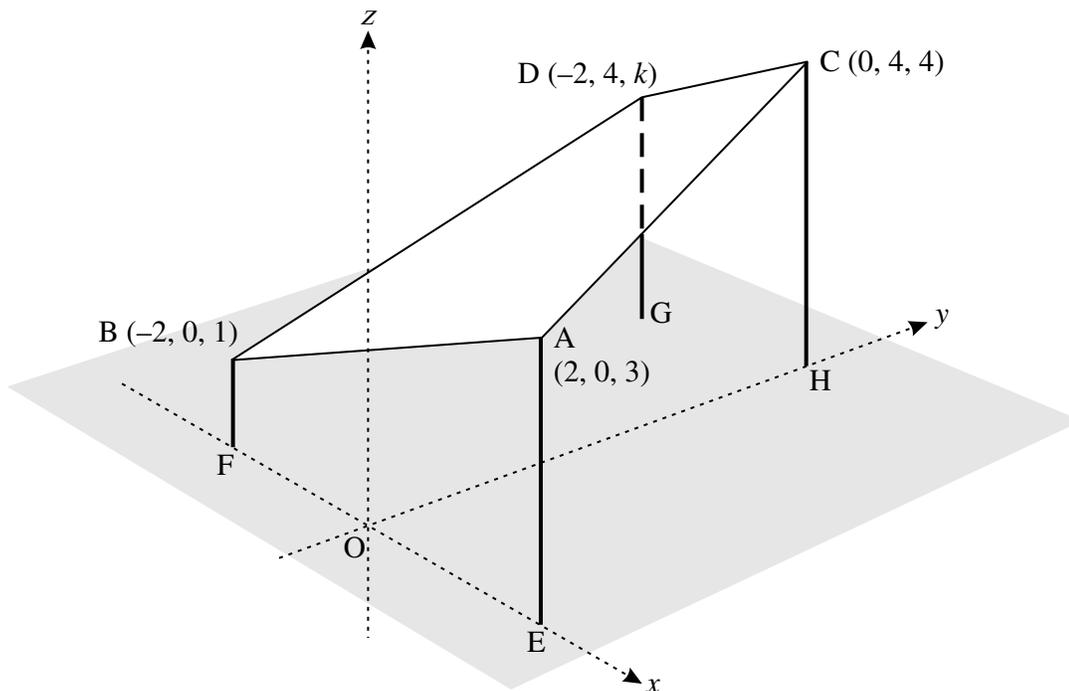


Fig. 7

- (i) Write down the vectors  $\vec{AB}$  and  $\vec{AC}$ . Hence calculate the angle BAC. [6]
- (ii) Verify that the equation of the plane ABC is  $x + y - 2z + d = 0$ , where  $d$  is a constant to be determined.
- Calculate the acute angle the plane makes with the horizontal plane. [7]
- (iii) Given that A, B, D and C are coplanar, show that  $k = 3$ .
- Hence show that ABDC is a trapezium, and find the ratio of CD to AB. [5]

- 3 A straight pipeline AB passes through a mountain. With respect to axes  $Oxyz$ , with  $Ox$  due East,  $Oy$  due North and  $Oz$  vertically upwards, A has coordinates  $(-200, 100, 0)$  and B has coordinates  $(100, 200, 100)$ , where units are metres.

(i) Verify that  $\overrightarrow{AB} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$  and find the length of the pipeline. [3]

- (ii) Write down a vector equation of the line AB, and calculate the angle it makes with the vertical. [6]

A thin flat layer of hard rock runs through the mountain. The equation of the plane containing this layer is  $x + 2y + 3z = 320$ .

- (iii) Find the coordinates of the point where the pipeline meets the layer of rock. [4]

- (iv) By calculating the angle between the line AB and the normal to the plane of the layer, find the angle at which the pipeline cuts through the layer. [5]

- 4 When a light ray passes from air to glass, it is deflected through an angle. The light ray ABC starts at point A (1, 2, 2), and enters a glass object at point B (0, 0, 2). The surface of the glass object is a plane with normal vector  $\mathbf{n}$ . Fig. 7 shows a cross-section of the glass object in the plane of the light ray and  $\mathbf{n}$ .

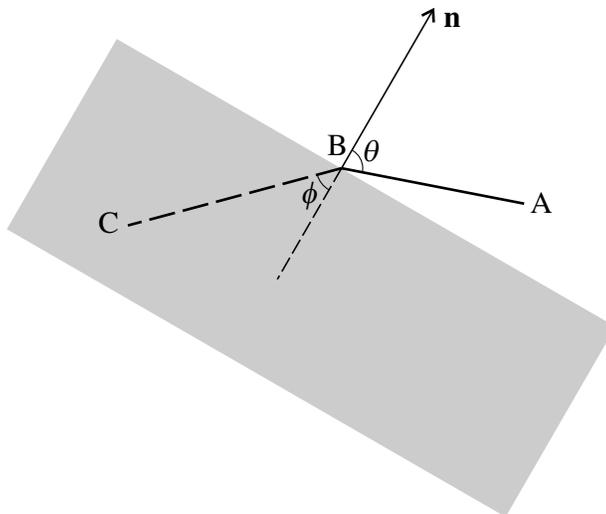


Fig. 7

- (i) Find the vector  $\overrightarrow{AB}$  and a vector equation of the line AB. [2]

The surface of the glass object is a plane with equation  $x + z = 2$ . AB makes an acute angle  $\theta$  with the normal to this plane.

- (ii) Write down the normal vector  $\mathbf{n}$ , and hence calculate  $\theta$ , giving your answer in degrees. [5]

The line BC has vector equation  $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$ . This line makes an acute angle  $\phi$  with the normal to the plane.

- (iii) Show that  $\phi = 45^\circ$ . [3]

- (iv) Snell's Law states that  $\sin \theta = k \sin \phi$ , where  $k$  is a constant called the refractive index. Find  $k$ . [2]

The light ray leaves the glass object through a plane with equation  $x + z = -1$ . Units are centimetres.

- (v) Find the point of intersection of the line BC with the plane  $x + z = -1$ . Hence find the distance the light ray travels through the glass object. [5]