

Trig. Compound Angle & Double Angle Formulae

Question Paper 1

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 4
Topic	Trigonometry
Sub Topic	Trig. Compound angle & Double angle Formulae
Booklet	Question Paper 1

Time Allowed: 70 minutes

Score: /58

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 Express $2 \sin \theta - 3 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants to be determined, and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the greatest and least possible values of $1 + 2 \sin \theta - 3 \cos \theta$. [6]

- 2 Express $4 \cos \theta - \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $4 \cos \theta - \sin \theta = 3$, for $0 \leq \theta \leq 2\pi$. [7]

- 3 Archimedes, about 2200 years ago, used regular polygons inside and outside circles to obtain approximations for π .

- (i) Fig. 8.1 shows a regular 12-sided polygon inscribed in a circle of radius 1 unit, centre O. AB is one of the sides of the polygon. C is the midpoint of AB. Archimedes used the fact that the circumference of the circle is greater than the perimeter of this polygon.

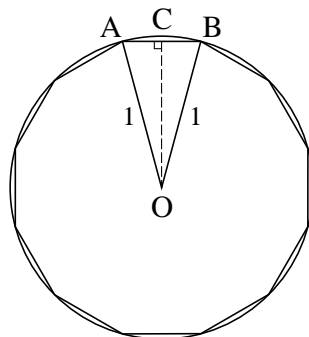


Fig. 8.1

- (A) Show that $AB = 2 \sin 15^\circ$. [2]

- (B) Use a double angle formula to express $\cos 30^\circ$ in terms of $\sin 15^\circ$. Using the exact value of $\cos 30^\circ$, show that $\sin 15^\circ = \frac{1}{2}\sqrt{2 - \sqrt{3}}$. [4]

- (C) Use this result to find an exact expression for the perimeter of the polygon.

Hence show that $\pi > 6\sqrt{2 - \sqrt{3}}$. [2]

- (ii) In Fig. 8.2, a regular 12-sided polygon lies outside the circle of radius 1 unit, which touches each side of the polygon. F is the midpoint of DE. Archimedes used the fact that the circumference of the circle is less than the perimeter of this polygon.

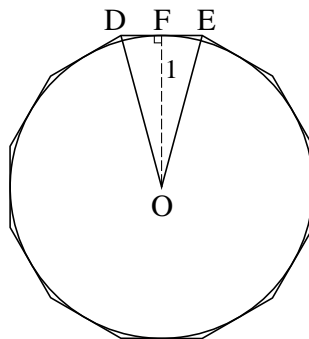


Fig. 8.2

- (A) Show that $DE = 2 \tan 15^\circ$. [2]

- (B) Let $t = \tan 15^\circ$. Use a double angle formula to express $\tan 30^\circ$ in terms of t .

Hence show that $t^2 + 2\sqrt{3}t - 1 = 0$. [3]

- (C) Solve this equation, and hence show that $\pi < 12(2 - \sqrt{3})$. [4]

- (iii) Use the results in parts (i)(C) and (ii)(C) to establish upper and lower bounds for the value of π , giving your answers in decimal form. [2]

4 Solve the equation $\cos 2\theta = \sin \theta$ for $0 \leq \theta \leq 2\pi$, giving your answers in terms of π . [7]

5 Express $\sqrt{3} \sin x - \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Express α in the form $k\pi$.

Find the exact coordinates of the maximum point of the curve $y = \sqrt{3} \sin x - \cos x$ for which $0 < x < 2\pi$. [6]

6 Express $\sin \theta - 3 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants to be determined, and $0^\circ < \alpha < 90^\circ$.

Hence solve the equation $\sin \theta - 3 \cos \theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$. [7]

7 Fig. 1 shows part of the graph of $y = \sin x - \sqrt{3} \cos x$.

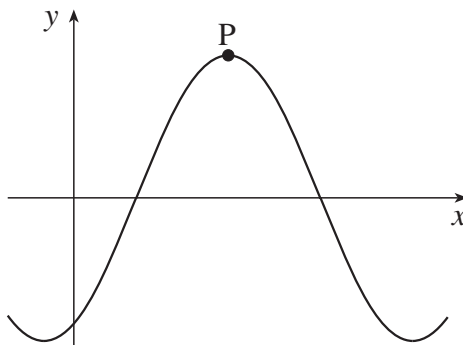


Fig. 1

Express $\sqrt{3} \sin x - \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 \leq \alpha \leq \frac{1}{2}\pi$.

Hence write down the exact coordinates of the turning point P. [6]