

The Poisson distribution

Question Paper 2

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	The Poisson distribution
Sub Topic	
Booklet	Question Paper 2

Time Allowed: 64 minutes

Score: /53

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The number of workers, X , absent from a factory on a particular day has the distribution $B(80, 0.01)$.

(i) Explain why it is appropriate to use a Poisson distribution as an approximating distribution for X . [2]

(ii) Use the Poisson distribution to find the probability that the number of workers absent during 12 randomly chosen days is more than 2 and less than 6. [3]

Following a change in working conditions, the management wishes to test whether the mean number of workers absent per day has decreased.

(iii) During 10 randomly chosen days, there were a total of 2 workers absent. Use the Poisson distribution to carry out the test at the 2% significance level. [5]

2 The random variable X has the distribution $Po(1.3)$. The random variable Y is defined by $Y = 2X$.

(i) Find the mean and variance of Y . [3]

(ii) Give a reason why the variable Y does not have a Poisson distribution. [1]

3 An engineering test consists of 100 multiple-choice questions. Each question has 5 suggested answers, only one of which is correct. Ashok knows nothing about engineering, but he claims that his general knowledge enables him to get more questions correct than just by guessing. Ashok actually gets 27 answers correct. Use a suitable approximating distribution to test at the 5% significance level whether his claim is justified [5]

- 4 Customers arrive at an enquiry desk at a constant average rate of 1 every 5 minutes.
- (i) State one condition for the number of customers arriving in a given period to be modelled by a Poisson distribution. [1]
- Assume now that a Poisson distribution is a suitable model.
- (ii) Find the probability that exactly 5 customers will arrive during a randomly chosen 30-minute period. [2]
- (iii) Find the probability that fewer than 3 customers will arrive during a randomly chosen 12-minute period. [3]
- (iv) Find an estimate of the probability that fewer than 30 customers will arrive during a randomly chosen 2-hour period. [4]
- 5 The numbers of men and women who visit a clinic each hour are independent Poisson variables with means 2.4 and 2.8 respectively.
- (i) Find the probability that, in a half-hour period,
- (a) 2 or more men and 1 or more women will visit the clinic, [4]
- (b) a total of 3 or more people will visit the clinic. [3]
- (ii) Find the probability that, in a 10-hour period, a total of more than 60 people will visit the clinic. [4]

- 6 A book contains 40 000 words. For each word, the probability that it is printed wrongly is 0.0001 and these errors occur independently. The number of words printed wrongly in the book is represented by the random variable X .
- (i) State the exact distribution of X , including the values of any parameters. [1]
 - (ii) State an approximate distribution for X , including the values of any parameters, and explain why this approximate distribution is appropriate. [3]
 - (iii) Use this approximate distribution to find the probability that there are more than 3 words printed wrongly in the book. [3]
- 7 A random variable has the distribution $Po(31)$. Name an appropriate approximating distribution and state the mean and standard deviation of this approximating distribution. [3]
- 8 2% of biscuits on a production line are broken. Broken biscuits occur randomly. 180 biscuits are checked to see whether they are broken. Use a suitable approximation to find the probability that fewer than 4 are broken. [3]