The Poisson distribution

Question Paper 3

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	The Poisson distribution
Sub Topic	
Booklet	Question Paper 3

Time Allowed: **64 minutes**

Score: **/53**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 An airline knows that some people who have bought tickets may not arrive for the f ight. The airline therefore sells more tickets than the number of seats that are available. For one f ight there are 210 seats available and 213 people have bought tickets. The probability of any person who has bought a ticket not arriving for the f ight is $\frac{1}{50}$.

(i) By considering the number of people who do **not** arrive for the f ight, use a suitable approximation to calculate the probability that more people will arrive than there are seats available. [4]

Independently, on another f ight for which 135 people have bought tickets, the probability of any person not arriving is $\frac{1}{75}$.

(ii) Calculate the probability that, for both these f ights, the total number of people who do not arrive is 5. [3]

2 In their football matches, Rovers score goals independently and at random times. Their average rate of scoring is 2.3 goals per match.

(i) State the expected number of goals that Rovers will score in the firs half of a match. [1]

(ii) Find the probability that Rovers will not score any goals in the f rst half of a match but will score one or more goals in the second half of the match. [2]

(iii) Football matches last for 90 minutes. In a particular match, Rovers score one goal in the fi st 30 minutes. Find the probability that they will score at least one further goal in the remaining 60 minutes. [3]

Independently of the number of goals scored by Rovers, the number of goals scored per football match by United has a Poisson distribution with mean 1.8.

(iv) Find the probability that a total of at least 3 goals will be scored in a particular match when Rovers play United. [3]

3 The random variable X denotes the number of worms on a one metre length of a country path after heavy rain. It is given that X has a Poisson distribution.

(i) For one particular path, the probability that $X = 2$ is three times the probability that $X = 4$. Find the probability that there are more than 3 worms on a 3.5 metre length of this path. [5]

(ii) For another path the mean of X is 1.3.

(a) On this path the probability that there is at least 1 worm on a length of k metres is 0.96. Find k. [4]

(b) Find the probability that there are more than 1250 worms on a one kilometre length of this path. [3]

4 In summer, wasps' nests occur randomly in the south of England at an average rate of 3 nests for every 500 houses.

(i) Find the probability that two villages in the south of England, with 600 houses and 700 houses, have a total of exactly 3 wasps' nests. [3]

(ii) Use a suitable approximation to estimate the probability of there being fewer than 369 wasps' nests in a town with 64 000 houses. [4]

5 The number of words on a page of a book can be modelled by a normal distribution with mean 403 and standard deviation 26.8. Find the probability that the average number of words per page in a random sample of 6 pages is less than 410. [4]

6 A shopkeeper sells electric fans. The demand for fans follows a Poisson distribution with mean 3.2 per week.

(i) Find the probability that the demand is exactly 2 fans in any one week. [2]

(ii) The shopkeeper has 4 fans in his shop at the beginning of a week. Find the probability that this will not be enough to satisfy the demand for fans in that week. [4]

(iii) Given instead that he has n fans in his shop at the beginning of a week, find by trial and error, the least value of n for which the probability of his not being able to satisfy the demand for fans in that week is less than 0.05. [4]

7 The number of radioactive particles emitted per second by a certain metal is random and has mean 1.7. The radioactive metal is placed next to an object which independently emits particles at random such that the mean number of particles emitted per second is 0.6. Find the probability that the total number of particles emitted in the next 3 seconds is 6, 7 or 8. [4]