

The Poisson distribution

Question Paper 4

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	The Poisson distribution
Sub Topic	
Booklet	Question Paper 4

Time Allowed: 64 minutes

Score: /53

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 A certain machine makes matches. One match in 10 000 on average is defective. Using a suitable approximation, calculate the probability that a random sample of 45 000 matches will include 2, 3 or 4 defective matches. [5]
- 2 The number of emergency telephone calls to the electricity board office in a certain area in t minutes is known to follow a Poisson distribution with mean $\frac{1}{80}t$.
- (i) Find the probability that there will be at least 3 emergency telephone calls to the office in any 20-minute period. [4]
- (ii) The probability that no emergency telephone call is made to the office in a period of k minutes is 0.9. Find k . [4]
- 3 1.5% of the population of the UK can be classified as ‘very tall’.
- (i) The random variable X denotes the number of people in a sample of n people who are classified as very tall. Given that $E(X) = 55$, find n . [2]
- (ii) By using the Poisson distribution as an approximation to a binomial distribution, calculate an approximate value for the probability that a sample of size 210 will contain fewer than 3 people who are classified as very tall. [3]
- 4 X and Y are independent random variables each having a Poisson distribution. X has mean 2.5 and Y has mean 3.1.
- (i) Find $P(X + Y > 3)$. [4]
- (ii) A random sample of 80 values of X is taken. Find the probability that the sample mean is less than 2.4. [4]

- 5 A publishing firm has found that errors in the first draft of a new book occur at random and that, on average, there is 1 error in every 3 pages of a first draft. Find the probability that in a particular first draft there are
- (i) exactly 2 errors in 10 pages, [2]
 - (ii) at least 3 errors in 6 pages, [3]
 - (iii) fewer than 50 errors in 200 pages. [4]
- 6 The independent random variables X and Y have standard deviations 3 and 6 respectively. Calculate the standard deviation of $4X - 5Y$. [3]
- 7 In a golf tournament, the number of times in a day that a ‘hole-in-one’ is scored is denoted by the variable X , which has a Poisson distribution with mean 0.15. Mr Crump offers to pay \$200 each time that a hole-in-one is scored during 5 days of play. Find the expectation and variance of the amount that Mr Crump pays. [5]
- 8 In a certain lottery, 10 500 tickets have been sold altogether and each ticket has a probability of 0.0002 of winning a prize. The random variable X denotes the number of prize-winning tickets that have been sold.
- (i) State, with a justification, an approximating distribution for X . [3]
 - (ii) Use your approximating distribution to find $P(X < 4)$. [3]
 - (iii) Use your approximating distribution to find the conditional probability that $X < 4$, given that $X \geq 1$. [4]