

The Poisson distribution

Question Paper 5

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	The Poisson distribution
Sub Topic	
Booklet	Question Paper 5

Time Allowed: 54 minutes

Score: /45

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 People arrive at a checkout in a store at random, and at a constant mean rate of 0.7 per minute. Find the probability that

(i) exactly 3 people arrive at the checkout during a 5-minute period, [2]

(ii) at least 30 people arrive at the checkout during a 1-hour period. [4]

People arrive independently at another checkout in the store at random, and at a constant mean rate of 0.5 per minute.

(iii) Find the probability that a total of more than 3 people arrive at this pair of checkouts during a 2-minute period. [4]

2 The masses, in grams, of apples of a certain type are normally distributed with mean 60.4 and standard deviation 8.2. The apples are packed in bags, with each bag containing 8 randomly chosen apples. The bags are checked by Quality Control and any bag containing apples with a total mass of less than 436 g is rejected. Find the proportion of bags that are rejected. [4]

3 The proportion of people who have a particular gene, on average, is 1 in 1000. A random sample of 3500 people in a certain country is chosen and the number of people, X , having the gene is found.

(i) State the distribution of X ; and state also an appropriate approximating distribution. Give the values of any parameters in each case. Justify your choice of the approximating distribution. [3]

(ii) Use the approximating distribution to find $P(X \leq 3)$. [2]

- 4 (i) The following tables show the probability distributions for the random variables V and W .

v	-1	0	1	>1	w	0	0.5	1	>1
$P(V = v)$	0.368	0.368	0.184	0.080	$P(W = w)$	0.368	0.368	0.184	0.080

For each of the variables V and W state how you can tell from its probability distribution that it does NOT have a Poisson distribution. [2]

- (ii) The random variable X has the distribution $Po(\lambda)$. It is given that

$$P(X = 0) = p \quad \text{and} \quad P(X = 1) = 2.5p,$$

where p is a constant.

- (a) Show that $\lambda = 2.5$. [1]
- (b) Find $P(X \geq 3)$. [2]
- (iii) The random variable Y has the distribution $Po(\mu)$, where $\mu > 30$. Using a suitable approximating distribution, it is found that $P(Y > 40) = 0.5793$ correct to 4 decimal places. Find μ . [5]

- 5 The number of calls per day to an enquiry desk has a Poisson distribution. In the past the mean has been 5. In order to test whether the mean has changed, the number of calls on a random sample of 10 days was recorded. The total number of calls was found to be 61. Use an approximate distribution to test at the 10% significance level whether the mean has changed. [5]

- 6 (i) The random variable W has the distribution $Po(1.5)$. Find the probability that the sum of 3 independent values of W is greater than 2. [3]
- (ii) The random variable X has the distribution $Po(\lambda)$. Given that $P(X = 0) = 0.523$, find the value of λ correct to 3 significant figures. [2]

- (iii) The random variable Y has the distribution $Po(\mu)$, where $\mu \neq 0$. Given that

$$P(Y = 3) = 24 \times P(Y = 1),$$

find μ . [3]

- 7 On average 1 in 25 000 people have a rare blood condition. Use a suitable approximating distribution to find the probability that fewer than 2 people in a random sample of 100 000 have the condition. [3]