

# The Poisson distribution

## Question Paper 6

<b>Level</b>	International A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	CIE
<b>Topic</b>	The Poisson distribution
<b>Sub Topic</b>	
<b>Booklet</b>	Question Paper 6

**Time Allowed:** 63 minutes

**Score:** /52

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A Lost Property office is open 7 days a week. It may be assumed that items are handed in to the office randomly, singly and independently.

(i) State another condition for the number of items handed in to have a Poisson distribution. [1]

It is now given that the number of items handed in per week has the distribution  $Po(4.0)$ .

(ii) Find the probability that exactly 2 items are handed in on a particular day. [2]

(iii) Find the probability that at least 4 items are handed in during a 10-day period. [3]

(iv) Find the probability that, during a certain week, 5 items are handed in altogether, but no items are handed in on the first day of the week. [3]

2 Weights of cups have a normal distribution with mean 91 g and standard deviation 3.2 g. Weights of saucers have an independent normal distribution with mean 72 g and standard deviation 2.6 g. Cups and saucers are chosen at random to be packed in boxes, with 6 cups and 6 saucers in each box. Given that each empty box weighs 550 g, find the probability that the total weight of a box containing 6 cups and 6 saucers exceeds 1550 g. [5]

3 The probability that a new car of a certain type has faulty brakes is 0.008. A random sample of 520 new cars of this type is chosen, and the number,  $X$ , having faulty brakes is noted.

(i) Describe fully the distribution of  $X$  and describe also a suitable approximating distribution. Justify this approximating distribution. [4]

(ii) Use your approximating distribution to find

(a)  $P(X > 3)$ , [2]

(b) the smallest value of  $n$  such that  $P(X = n) > P(X = n + 1)$ . [3]

- 4 It is known that 1.2% of rods made by a certain machine are bent. The random variable  $X$  denotes the number of bent rods in a random sample of 400 rods.
- (i) State the distribution of  $X$ . [2]
  - (ii) State, with a reason, a suitable approximate distribution for  $X$ . [2]
  - (iii) Use your approximate distribution to find the probability that the sample will include more than 2 bent rods. [2]
- 5 Packets of cereal are packed in boxes, each containing 6 packets. The masses of the packets are normally distributed with mean 510 g and standard deviation 12 g. The masses of the empty boxes are normally distributed with mean 70 g and standard deviation 4 g.
- (i) Find the probability that the total mass of a full box containing 6 packets is between 3050 g and 3150 g. [5]
  - (ii) A packet and an empty box are chosen at random. Find the probability that the mass of the packet is at least 8 times the mass of the empty box. [5]
- 6 The independent random variables  $X$  and  $Y$  have the distributions  $Po(2)$  and  $Po(3)$  respectively.
- (i) Given that  $X + Y = 5$ , find the probability that  $X = 1$  and  $Y = 4$ . [4]
  - (ii) Given that  $P(X = r) = \frac{2}{3}P(X = 0)$ , show that  $3 \times 2^{r-1} = r!$  and verify that  $r = 4$  satisfies this equation. [2]
- 7 Calls arrive at a helpdesk randomly and at a constant average rate of 1.4 calls per hour. Calculate the probability that there will be
- (i) more than 3 calls in  $2\frac{1}{2}$  hours, [3]
  - (ii) fewer than 1000 calls in four weeks (672 hours). [4]