

# The Poisson distribution

## Question Paper 8

<b>Level</b>	International A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	CIE
<b>Topic</b>	The Poisson distribution
<b>Sub Topic</b>	
<b>Booklet</b>	Question Paper 8

**Time Allowed:** 77 minutes

**Score:** /64

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1  $X$  is a random variable having the distribution  $B(12, \frac{1}{4})$ . A random sample of 60 values of  $X$  is taken. Find the probability that the sample mean is less than  $\bar{2.8}$ . [5]
- 2 The number of goals scored per match by Everly Rovers is represented by the random variable  $X$  which has mean 1.8.
- (i) State two conditions for  $X$  to be modelled by a Poisson distribution. [2]
- Assume now that  $X \sim \text{Po}(1.8)$ .
- (ii) Find  $P(2 < X < 6)$ . [2]
- (iii) The manager promises the team a bonus if they score at least 1 goal in each of the next 10 matches. Find the probability that they win the bonus. [4]
- 3 The number of adult customers arriving in a shop during a 5-minute period is modelled by a random variable with distribution  $\text{Po}(6)$ . The number of child customers arriving in the same shop during a 10-minute period is modelled by an independent random variable with distribution  $\text{Po}(4.5)$ .
- (i) Find the probability that during a randomly chosen 2-minute period, the total number of adult and child customers who arrive in the shop is less than 3. [3]
- (ii) During a sale, the manager claims that more adult customers are arriving than usual. In a randomly selected 30-minute period during the sale, 49 adult customers arrive. Test the manager's claim at the 2.5% significance level. [6]
- 4 A hotel kitchen has two dish-washing machines. The numbers of breakdowns per year by the two machines have independent Poisson distributions with means 0.7 and 1.0. Find the probability that the total number of breakdowns by the two machines during the next two years will be less than 3. [4]

- 5 On average, 1 in 2500 people have a particular gene.
- (i) Use a suitable approximation to find the probability that, in a random sample of 10 000 people, more than 3 people have this gene. [4]
  - (ii) The probability that, in a random sample of  $n$  people, none of them has the gene is less than 0.01. Find the smallest possible value of  $n$ . [3]
- 6 The weekly distance in kilometres driven by Mr Parry has a normal distribution with mean 512 and standard deviation 62. Independently, the weekly distance in kilometres driven by Mrs Parry has a normal distribution with mean 89 and standard deviation 7.4.
- (i) Find the probability that, in a randomly chosen week, Mr Parry drives more than 5 times as far as Mrs Parry. [5]
  - (ii) Find the mean and standard deviation of the total of the weekly distances in miles driven by Mr Parry and Mrs Parry. Use the approximation 8 kilometres = 5 miles. [3]
- 7 In restaurant  $A$  an average of 2.2% of tablecloths are stained and, independently, in restaurant  $B$  an average of 5.8% of tablecloths are stained.
- (i) Random samples of 55 tablecloths are taken from each restaurant. Use a suitable Poisson approximation to find the probability that a total of more than 2 tablecloths are stained. [4]
  - (ii) Random samples of  $n$  tablecloths are taken from each restaurant. The probability that at least one tablecloth is stained is greater than 0.99. Find the least possible value of  $n$ . [4]
- 8 A clinic deals only with flu vaccinations. The number of patients arriving every 15 minutes is modelled by the random variable  $X$  with distribution  $Po(4.2)$ .
- (i) State two assumptions required for the Poisson model to be valid. [2]
  - (ii) Find the probability that
    - (a) at least 1 patient will arrive in a 15-minute period, [2]
    - (b) fewer than 4 patients will arrive in a 10-minute period. [3]
  - (iii) The clinic is open for 20 hours each week. At the beginning of one week the clinic has enough vaccine for 370 patients. Use a suitable approximation to find the probability that this will not be enough vaccine for that week. [4]

- 9 In Europe the diameters of women's rings have mean 18.5 mm. Researchers claim that women in Jakarta have smaller fingers than women in Europe. The researchers took a random sample of 20 women in Jakarta and measured the diameters of their rings. The mean diameter was found to be 18.1 mm. Assuming that the diameters of women's rings in Jakarta have a normal distribution with standard deviation 1.1 mm, carry out a hypothesis test at the  $2\frac{1}{2}\%$  level to determine whether the researchers' claim is justified [5]