

The Poisson distribution

Question Paper 9

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	The Poisson distribution
Sub Topic	
Booklet	Question Paper 9

Time Allowed: 89 minutes

Score: /74

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 Major avalanches can be regarded as randomly occurring events. They occur at a uniform average rate of 8 per year.

(i) Find the probability that more than 3 major avalanches occur in a 3-month period. [3]

(ii) Find the probability that any two separate 4-month periods have a total of 7 major avalanches. [3]

(iii) Find the probability that a total of fewer than 137 major avalanches occur in a 20-year period. [4]

2 People arrive randomly and independently at the elevator in a block of flat at an average rate of 4 people every 5 minutes.

(i) Find the probability that exactly two people arrive in a 1-minute period. [2]

(ii) Find the probability that nobody arrives in a 15-second period. [2]

(iii) The probability that at least one person arrives in the next t minutes is 0.9. Find the value of t . [4]

3 The random variable X has the distribution $B(10, 0.15)$. Find the probability that the mean of a random sample of 50 observations of X is greater than 1.4. [5]

4 It is proposed to model the number of people per hour calling a car breakdown service between the times 09 00 and 21 00 by a Poisson distribution.

(i) Explain why a Poisson distribution may be appropriate for this situation. [2]

People call the car breakdown service at an average rate of 20 per hour, and a Poisson distribution may be assumed to be a suitable model.

(ii) Find the probability that exactly 8 people call in any half hour. [2]

(iii) By using a suitable approximation, find the probability that exactly 250 people call in the 12 hours between 09 00 and 21 00. [4]

- 5 A mathematics module is assessed by an examination and by coursework. The examination makes up 75% of the total assessment and the coursework makes up 25%. Examination marks, X , are distributed with mean 53.2 and standard deviation 9.3. Coursework marks, Y , are distributed with mean 78.0 and standard deviation 5.1. Examination marks and coursework marks are independent. Find the mean and standard deviation of the combined mark $0.75X + 0.25Y$. [4]
- 6 A certain make of washing machine has a wash-time with mean 56.9 minutes and standard deviation 4.8 minutes. A certain make of tumble dryer has a drying-time with mean 61.1 minutes and standard deviation 6.3 minutes. Both times are normally distributed and are independent of each other. Find the probability that a randomly chosen wash-time differs by more than 3 minutes from a randomly chosen drying-time. [6]
- 7 A dressmaker makes dresses for Easifit Fashions. Each dress requires 2.5 m^2 of material. Faults occur randomly in the material at an average rate of 4.8 per 20 m^2 .
- (i) Find the probability that a randomly chosen dress contains at least 2 faults. [3]
- Each dress has a belt attached to it to make an outfit. Independently of faults in the material, the probability that a belt is faulty is 0.03. Find the probability that, in an outfit,
- (ii) neither the dress nor its belt is faulty, [2]
- (iii) the dress has at least one fault and its belt is faulty. [2]
- The dressmaker attaches 300 randomly chosen belts to 300 randomly chosen dresses. An outfit in which the dress has at least one fault and its belt is faulty is rejected.
- (iv) Use a suitable approximation to find the probability that fewer than 3 outfits are rejected. [3]

- 8** A clock contains 4 new batteries each of which gives a voltage which is normally distributed with mean 1.54 volts and standard deviation 0.05 volts. The voltages of the batteries are independent. The clock will only work if the total voltage is greater than 5.95 volts.
- (i) Find the probability that the clock will work. [4]
- (ii) Find the probability that the average total voltage of the batteries of 20 clocks chosen at random exceeds 6.2 volts. [3]
- 9** In athletics matches the triple jump event consists of a hop, followed by a step, followed by a jump. The lengths covered by Albert in each part are independent normal variables with means 3.5 m, 2.9 m, 3.1 m and standard deviations 0.3 m, 0.25 m, 0.35 m respectively. The length of the triple jump is the sum of the three parts.
- (i) Find the mean and standard deviation of the length of Albert's triple jumps. [3]
- (ii) Find the probability that the mean of Albert's next four triple jumps is greater than 9 m. [3]
- 10** Machine *A* fills bags of fertiliser so that their weights follow a normal distribution with mean 20.05 kg and standard deviation 0.15 kg. Machine *B* fills bags of fertiliser so that their weights follow a normal distribution with mean 20.05 kg and standard deviation 0.27 kg.
- (i) Find the probability that the total weight of a random sample of 20 bags filled by machine *A* is at least 2 kg more than the total weight of a random sample of 20 bags filled by machine *B*. [6]
- (ii) A random sample of n bags filled by machine *A* is taken. The probability that the sample mean weight of the bags is greater than 20.07 kg is denoted by p . Find the value of n , given that $p = 0.0250$ correct to 4 decimal places. [4]