Linear combinations for random variables

Question Paper 2

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Linear combinations for random variables
Sub Topic	
Booklet	Question Paper 2

Time Allowed: **59 minutes**

Score: **/49**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 Three coats of paint are sprayed onto a surface. The thicknesses, in millimetres, of the three coats have independent distributions $N(0.13, 0.02^2)$, $N(0.14, 0.03^2)$ and $N(0.10, 0.01^2)$. Find the probability that, at a randomly chosen place on the surface, the total thickness of the three coats of paint is less than 0.30 millimetres. [5]

2 Test scores, X, have mean 54 and variance 144. The scores are scaled using the formula $Y = a + bX$, where a and b are constants and $b > 0$. The scaled scores, Y, have mean 50 and variance 100. Find the values of a and b. [4]

3 Ranjit goes to mathematics lectures and physics lectures. The length, in minutes, of a mathematics lecture is modelled by the variable X with distribution $N(36, 3.5^2)$. The length, in minutes, of a physics lecture is modelled by the independent variable Y with distribution $N(55, 5.2^2)$.

(i) Find the probability that the total length of two mathematics lectures and one physics lecture is less than 140 minutes. [4]

(ii) Ranjit calculates how long he will need to spend revising the content of each lecture as follows. Each minute of a mathematics lecture requires 1 minute of revision and each minute of a physics lecture requires $1\frac{1}{2}$ minutes of revision. Find the probability that the total revision time required for one mathematics lecture and one physics lecture is more than 100 minutes. [4]

4 The marks of candidates in Mathematics and English in 2009 were represented by the independent random variables X and Y with distributions $N(28, 5.6^2)$ and $N(52, 12.4^2)$ respectively. Each candidate's marks were combined to give a final mark F, where $F = X + \frac{1}{2}Y$.

(i) Find $E(F)$ and $Var(F)$. [3]

(ii) The final marks of a random sample of 10 candidates from Grinford in 2009 had a mean of 49. Test at the 5% significance level whether this result suggests that the mean final mark of all candidates from Grinford in 2009 was lower than elsewhere. [5]

5 The masses, in milligrams, of three minerals found in 1 tonne of a certain kind of rock are modelled by three independent random variables P, Q and R, where $P \sim N(46, 19^2)$, $Q \sim N(53, 23^2)$ and $R \sim N(25, 10^2)$. The total value of the minerals found in 1 tonne of rock is modelled by the random variable V, where $V = P + Q + 2R$. Use the model to fin the probability of findin minerals with a value of at least 93 in a randomly chosen tonne of rock. [7]

6 A clinic monitors the amount, X milligrams per litre, of a certain chemical in the blood stream of patients. For patients who are taking drug A, it has been found that the mean value of X is 0.336. A random sample of 100 patients taking a new drug, B, was selected and the values of X were found. The results are summarised below.

$$n = 100, \qquad \Sigma x = 43.5, \qquad \Sigma x^2 = 31.56.$$

(i) Test at the 1% signif cance level whether the mean amount of the chemical in the blood stream of patients taking drug B is different from that of patients taking drug A. [8]

(ii) For the test to be valid, is it necessary to assume a normal distribution for the amount of chemical in the blood stream of patients taking drug B? Justify your answer. [2]

7 The weights of pebbles on a beach are normally distributed with mean 48.5 grams and standard deviation 12.4 grams.

(i) Find the probability that the mean weight of a random sample of 5 pebbles is greater than 51 grams. [3]

(ii) The probability that the mean weight of a random sample of n pebbles is less than 51.6 grams is 0.9332. Find the value of n. [4]