

# Linear combinations for random variables

## Question Paper 3

<b>Level</b>	International A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	CIE
<b>Topic</b>	Linear combinations for random variables
<b>Sub Topic</b>	
<b>Booklet</b>	Question Paper 3

**Time Allowed:** 74 minutes

**Score:** /61

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1** The volume of liquid in cans of cola is normally distributed with mean 330 millilitres and standard deviation 5.2 millilitres. The volume of liquid in bottles of tonic water is normally distributed with mean 500 millilitres and standard deviation 7.1 millilitres.
- (i) Find the probability that 3 randomly chosen cans of cola contain less liquid than 2 randomly chosen bottles of tonic water. [5]
  - (ii) A new drink is made by mixing the contents of 2 cans of cola with half a bottle of tonic water. Find the probability that the volume of the new drink is more than 900 millilitres. [4]
- 2** The masses of packets of cornflakes are normally distributed with standard deviation 11 g. A random sample of 20 packets was weighed and found to have a mean mass of 746 g.
- (i) Test at the 4% significance level whether there is enough evidence to conclude that the population mean mass is less than 750 g. [4]
  - (ii) Given that the population mean mass actually is 750 g, find the smallest possible sample size,  $n$ , for which it is at least 97% certain that the mean mass of the sample exceeds 745 g. [4]
- 3** (a) Random variables  $Y$  and  $X$  are related by  $Y = a + bX$ , where  $a$  and  $b$  are constants and  $b > 0$ . The standard deviation of  $Y$  is twice the standard deviation of  $X$ . The mean of  $Y$  is 7.92 and is 0.8 more than the mean of  $X$ . Find the values of  $a$  and  $b$ . [3]
- (b) Random variables  $R$  and  $S$  are such that  $R \sim N(\mu, 2^2)$  and  $S \sim N(2\mu, 3^2)$ . It is given that  $P(R + S > 1) = 0.9$ .
- (i) Find  $\mu$ . [4]
  - (ii) Hence find  $P(S > R)$ . [3]

- 4 Weights of garden tables are normally distributed with mean 36 kg and standard deviation 1.6 kg. Weights of garden chairs are normally distributed with mean 7.3 kg and standard deviation 0.4 kg. Find the probability that the total weight of 2 randomly chosen tables is more than the total weight of 10 randomly chosen chairs. [5]

- 5 Climbing ropes produced by a manufacturer have breaking strengths which are normally distributed with mean 160 kg and standard deviation 11.3 kg. A group of climbers have weights which are normally distributed with mean 66.3 kg and standard deviation 7.1 kg.

- (i) Find the probability that a rope chosen randomly will break under the combined weight of 2 climbers chosen randomly. [5]

Each climber carries, in a rucksack, equipment amounting to half his own weight.

- (ii) Find the mean and variance of the combined weight of a climber and his rucksack. [3]

- (iii) Find the probability that the combined weight of a climber and his rucksack is greater than 87 kg. [2]

- 6 The weights of men follow a normal distribution with mean 71 kg and standard deviation 7 kg. The weights of women follow a normal distribution with mean 57 kg and standard deviation 5 kg. The total weight of 5 men and 2 women chosen randomly is denoted by  $X$  kg.

- (i) Show that  $E(X) = 469$  and  $\text{Var}(X) = 295$ . [2]

- (ii) The total weight of 4 men and 3 women chosen randomly is denoted by  $Y$  kg. Find the mean and standard deviation of  $X - Y$  and hence find  $P(X - Y > 22)$ . [5]

- 7 Of people who wear contact lenses, 1 in 1500 on average have laser treatment for short sight.
- (i) Use a suitable approximation to find the probability that, of a random sample of 2700 contact lens wearers, more than 2 people have laser treatment. [4]
  - (ii) In a random sample of  $n$  contact lens wearers the probability that no one has laser treatment is less than 0.01. Find the least possible value of  $n$ . [3]
- 8 Tien throws a ball. The distance it travels can be modelled by a normal distribution with mean 20 m and variance  $9 \text{ m}^2$ . His younger sister Su Chen also throws a ball and the distance her ball travels can be modelled by a normal distribution with mean 14 m and variance  $12 \text{ m}^2$ . Su Chen is allowed to add 5 metres on to her distance and call it her ‘upgraded distance’. Find the probability that Tien’s distance is larger than Su Chen’s upgraded distance. [5]