Linear combinations for random variables

Question Paper 4

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Linear combinations for random variables
Sub Topic	
Booklet	Question Paper 4

Time Allowed: **75 minutes**

Score: **/62**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The distance driven in a week by a long-distance lorry driver is a normally distributed random variable with mean 1850 km and standard deviation 117 km.

(i) Find the probability that in a random sample of 26 weeks his average distance driven per week is more than 1800 km. [3]

(ii) New driving regulations are introduced and in a random sample of 26 weeks after their introduction the lorry driver drives a total of 47 658 km. Assuming the standard deviation remains unchanged, test at the 10% level whether his mean weekly driving distance has changed. [5]

2 Bottles of wine are stacked in racks of 12. The weights of these bottles are normally distributed with mean 1.3 kg and standard deviation 0.06 kg. The weights of the empty racks are normally distributed with mean 2 kg and standard deviation 0.3 kg.

(i) Find the probability that the total weight of a full rack of 12 bottles of wine is between 17 kg and 18 kg. [5]

(ii) Two bottles of wine are chosen at random. Find the probability that they differ in weight by more than 0.05 kg. [5]

3 The independent variables X and Y are such that $X \sim \mathrm{B}(10, 0.8)$ and $Y \sim \mathrm{Po}(3)$. Find

(i) $\mathrm{E}(7X + 5Y - 2)$, [2]

(ii) $\mathrm{Var}(4X - 3Y + 3)$, [4]

(iii) $\mathrm{P}(2X - Y = 18)$. [4]

4 Each day Samuel travels from A to B and from B to C. He then returns directly from C to A. The times, in minutes, for these three journeys have the independent distributions $N(20, 2^2)$, $N(18, 1.5^2)$ and $N(30, 1.8^2)$, respectively. Find the probability that, on a randomly chosen day, the total time for his two journeys from A to B and B to C is less than the time for his return journey from C to A. [5]

5 In an examination, the marks in the theory paper and the marks in the practical paper are denoted by the random variables ; d respectively, where ; ~ N(57, 13) and <~ N(28, 5). You may assume that each candidate's marks in the two papers are independent. The fina score of each candidate is found by calculating ; + 2.5 ≤ A candidate is chosen at random. Without using a continuity correction, fin the probability that this candidate

(i) has a f nal score that is greater than 140, [5]

(ii) obtains at least 20 more marks in the theory paper than in the practical paper. [5]

6 The mean and variance of the random variable X are 5.8 and 3.1 respectively. The random variable S is the sum of three independent values of X. The independent random variable T is defin d by $T = 3X + 2$.

(i) Find the variance of S. [1]

(ii) Find the variance of T. [1]

(iii) Find the mean and variance of $S - T$. [3]

7 An examination consists of a written paper and a practical test. The written paper marks (M) have mean 54.8 and standard deviation 16.0. The practical test marks (P) are independent of the written paper marks and have mean 82.4 and standard deviation 4.8. The fina mark is found by adding 75% of M to 25% of P. Find the mean and standard deviation of the fina marks for the examination. [3]

8 At work Jerry receives emails randomly at a constant average rate of 15 emails per hour.

(i) Find the probability that Jerry receives more than 2 emails during a 20-minute period at work. [3]

(ii) Jerry's working day is 8 hours long. Find the probability that Jerry receives fewer than 110 emails per day on each of 2 working days. [4]

(iii) At work Jerry also receives texts randomly and independently at a constant average rate of 1 text every 10 minutes. Find the probability that the total number of emails and texts that Jerry receives during a 5-minute period at work is more than 2 and less than 6. [4]