# Continuous random variables

## Question Paper 1

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|---|---|
| **Level** | International A Level |
| **Subject** | Maths |
| **Exam Board** | CIE |
| **Topic** | Continuous random variables |
| **Sub Topic** | |
| **Booklet** | Question Paper 1 |

**Time Allowed:** **60 minutes**

**Score:** **/50**

**Percentage:** **/100**

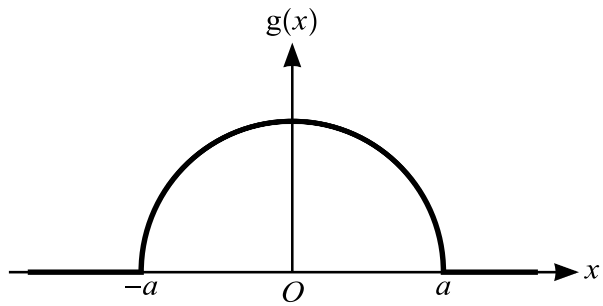
**Grade Boundaries:**

| A* | A | B | C | D | E | U |
|---|---|---|---|---|---|---|
| >85% | '77.5% | 70% | 62.5% | 57.5% | 45% | <45% |

**1** **(a)** The time for which Lucy has to wait at a certain traff c light each day is $T$ minutes, where $T$ has probability density function given by

$$f(t) = \begin{cases} \frac{3}{2}t - \frac{3}{4}t^2 & 0 \leqslant t \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

**(b)** Find the probability that, on a randomly chosen day, Lucy has to wait for less than half a minute at the traff c light. [3]



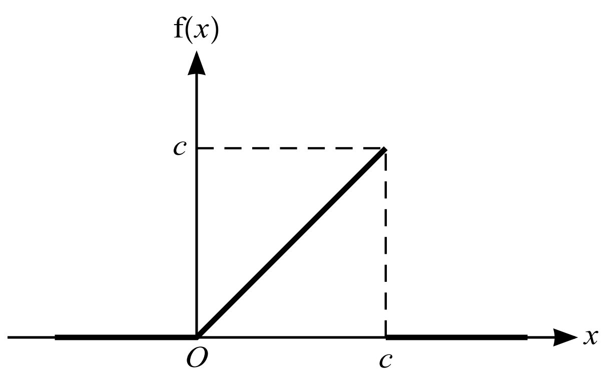

The diagram shows the graph of the probability density function, g, of a random variable $X$. The graph of g is a semicircle with centre $(0, 0)$ and radius $a$. Elsewhere $g(x) = 0$.

**(i)** Find the value of $a$. [2]

**(ii)** State the value of $E(X)$. [1]

**(iii)** Given that $P(X < -c) = 0.2$, f nd $P(X < c)$. [2]

**2**




The diagram shows the graph of the probability density function, f, of a random variable $X$.

**(i)** Find the value of the constant $c$. [2]

**(ii)** Find the value of $a$ such that $P(a < X < 1) = 0.1$. [4]

**(iii)** Find $E(X)$. [2]

**3** The volume, in $cm^3$, of liquid left in a glass by people when they have fini hed drinking all they want is modelled by the random variable $X$ with probability density function given by

$$f(x) = \begin{cases} k(x-2)^2 & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where $k$ is a constant.

**(i)** Show that $k = \frac{3}{8}$. [2]

**(ii)** 20% of people leave at least $d$ $cm^3$ of liquid in a glass. Find $d$. [3]

**(iii)** Find $E(X)$. [3]

**4** A random sample of 80 values of a variable $X$ is taken and these values are summarised below.

$$n = 80 \quad \Sigma x = 150.2 \quad \Sigma x^2 = 820.24$$

Calculate unbiased estimates of the population mean and variance of $X$ and hence fin a 95% confidenc interval for the population mean of $X$. [6]

**5** A fair six-sided die has faces numbered 1, 2, 3, 4, 5, 6. The score on one throw is denoted by $X$.

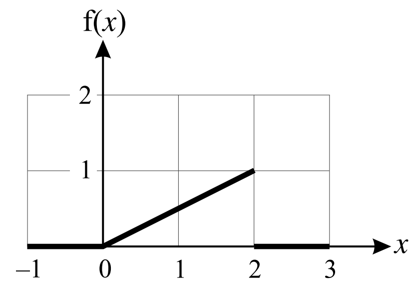
**(i)** Write down the value of $E(X)$ and show that $\text{Var}(X) = \frac{35}{12}$. [2]

Fayez has a six-sided die with faces numbered 1, 2, 3, 4, 5, 6. He suspects that it is biased so that when it is thrown it is more likely to show a low number than a high number. In order to test his suspicion, he plans to throw the die 50 times. If the mean score is less than 3 he will conclude that the die is biased.

**(ii)** Find the probability of a Type I error. [5]

**(iii)** With reference to this context, describe circumstances in which Fayez would make a Type II error. [2]

**6**




The diagram shows the graph of the probability density function, f, of a random variable $X$. Find the median of $X$. [3]

**7** A random variable $X$ has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x-1} & 3 \leqslant x \leqslant 5, \\ 0 & \text{otherwise,} \end{cases}$$

where $k$ is a constant.

**(i)** Show that $k = \dfrac{1}{\ln 2}$. [4]

**(ii)** Find $a$ such that $P(X < a) = 0.75$. [4]