

# Continuous random variables

## Question Paper 2

<b>Level</b>	International A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	CIE
<b>Topic</b>	Continuous random variables
<b>Sub Topic</b>	
<b>Booklet</b>	Question Paper 2

**Time Allowed:** 59 minutes

**Score:** /49

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 Darts are thrown at random at a circular board. The darts hit the board at distances  $X$  centimetres from the centre, where  $X$  is a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{2}{a^2}x & 0 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where  $a$  is a positive constant.

- (i) Verify that  $f$  is a probability density function whatever the value of  $a$ . [3]

It is now given that  $E(X) = 8$ .

- (ii) Find the value of  $a$ . [3]

- (iii) Find the probability that a dart lands more than 6 cm from the centre of the board. [3]

2

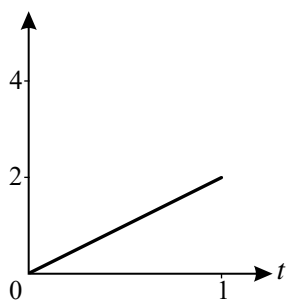


Fig. 1

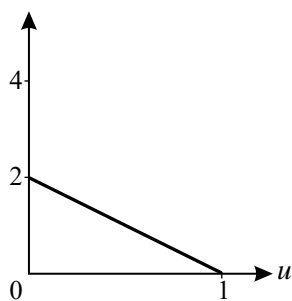


Fig. 2

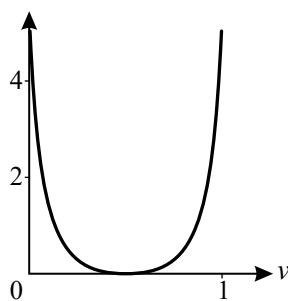


Fig. 3

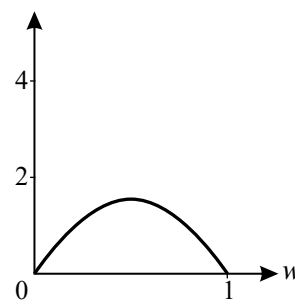


Fig. 4

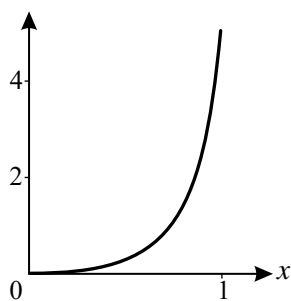


Fig. 5

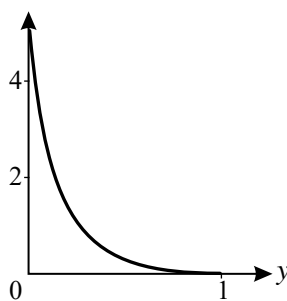


Fig. 6

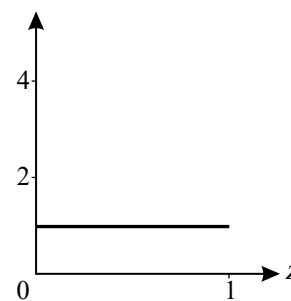


Fig. 7

Each of the random variables  $T$ ,  $U$ ,  $V$ ,  $W$ ,  $X$ ,  $Y$  and  $Z$  takes values between 0 and 1 only. Their probability density functions are shown in Figs 1 to 7 respectively.

(i) (a) Which of these variables has the largest median? [1]

(b) Which of these variables has the largest standard deviation? Explain your answer. [2]

(ii) Use Fig. 2 to find  $P(U < 0.5)$ . [2]

(iii) The probability density function of  $X$  is given by

$$f(x) = \begin{cases} ax^n & 0 \leq x \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where  $a$  and  $n$  are positive constants.

(a) Show that  $a = n + 1$ . [3]

(b) Given that  $E(X) = \frac{5}{6}$ , find  $a$  and  $n$ . [4]

- 3 The random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} ke^{-x} & 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that  $k = \frac{e}{e-1}$ . [3]

(ii) Find  $E(X)$  in terms of  $e$ . [4]

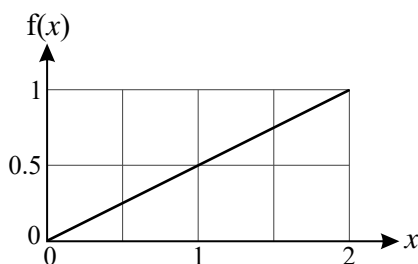
- 4 People arrive randomly and independently at a supermarket checkout at an average rate of 2 people every 3 minutes.

(i) Find the probability that exactly 4 people arrive in a 5-minute period. [2]

At another checkout in the same supermarket, people arrive randomly and independently at an average rate of 1 person each minute.

(ii) Find the probability that a total of fewer than 3 people arrive at the two checkouts in a 3-minute period. [3]

5



The diagram shows the graph of the probability density function,  $f$ , of a random variable  $X$  which takes values between 0 and 2 only.

(i) Find  $P(1 < X < 1.5)$ . [2]

(ii) Find the median of  $X$ . [3]

(iii) Find  $E(X)$ . [2]

- 6 A continuous random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} \frac{1}{6}x & 2 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

- (i) Find  $E(X)$ . [3]
- (ii) Find the median of  $X$ . [3]
- (iii) Two independent values of  $X$  are chosen at random. Find the probability that both these values are greater than 3. [3]