

Continuous random variables

Question Paper 3

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Continuous random variables
Sub Topic	
Booklet	Question Paper 3

Time Allowed: 68 minutes

Score: /56

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} k \cos x & 0 \leq x \leq \frac{1}{4}\pi, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \sqrt{2}$. [2]

(ii) Find $P(X > 0.4)$. [2]

(iii) Find the upper quartile of X . [3]

(iv) Find the probability that exactly 3 out of 5 random observations of X have values greater than the upper quartile. [2]

- 2 A computer user finds that unwanted emails arrive randomly at a uniform average rate of 1.27 per hour.

(i) Find the probability that more than 1 unwanted email arrives in a period of 5 hours. [2]

(ii) Find the probability that more than 850 unwanted emails arrive in a period of 700 hours. [4]

- 3 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{3}x(k-x) & 1 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that the value of k is $\frac{32}{9}$. [3]

(ii) Find $E(X)$. [2]

(iii) Is the median less than or greater than 1.5? Justify your answer numerically. [3]

- 4 The time in hours taken for clothes to dry can be modelled by the continuous random variable with probability density function given by

$$f(t) = \begin{cases} k\sqrt{t} & 1 \leq t \leq 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (i) Show that $k = \frac{3}{14}$. [3]
- (ii) Find the mean time taken for clothes to dry. [4]
- (iii) Find the median time taken for clothes to dry. [3]
- (iv) Find the probability that the time taken for clothes to dry is between the mean time and the median time. [2]

- 5 The length, X cm, of a piece of wooden planking is a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{1}{b} & 0 \leq x \leq b, \\ 0 & \text{otherwise,} \end{cases}$$

where b is a positive constant.

- (i) Find the mean and variance of X in terms of b . [3]

The lengths of a random sample of 100 pieces were measured and it was found that $\Sigma x = 950$.

- (ii) Show that the value of b estimated from this information is 19. [2]

Using this value of b ,

- (iii) find the probability that the length of a randomly chosen piece is greater than 11 cm, [1]
- (iv) find the probability that the mean length of a random sample of 336 pieces is less than 9 cm. [4]

- 6 At a town centre car park the length of stay in hours is denoted by the random variable X , which has probability density function given by

$$f(x) = \begin{cases} kx^{-\frac{3}{2}} & 1 \leq x \leq 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (i) Interpret the inequalities $1 \leq x \leq 9$ in the definition of $f(x)$ in the context of the question. [1]
- (ii) Show that $k = \frac{3}{4}$. [2]
- (iii) Calculate the mean length of stay. [3]

The charge for a length of stay of x hours is $(1 - e^{-x})$ dollars.

- (iv) Find the length of stay for the charge to be at least 0.75 dollars [3]
- (v) Find the probability of the charge being at least 0.75 dollars. [2]