

Continuous random variables

Question Paper 5

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Continuous random variables
Sub Topic	
Booklet	Question Paper 5

Time Allowed: 60 minutes

Score: /50

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 The probability density function of the random variable X is given by

$$f(x) = \begin{cases} \frac{3}{4}x(c-x) & 0 \leq x \leq c, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

(i) Show that $c = 2$. [3]

(ii) Sketch the graph of $y = f(x)$ and state the median of X . [3]

(iii) Find $P(X < 1.5)$. [4]

(iv) Hence write down the value of $P(0.5 < X < 1)$. [1]

- 2 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x} & 1 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(i) Show that $k = \frac{1}{\ln a}$. [3]

(ii) Find $E(X)$ in terms of a . [3]

(iii) Find the median of X in terms of a . [4]

- 3 The time, T hours, spent by people on a visit to a museum has probability density function

$$f(t) = \begin{cases} kt(16-t^2) & 0 \leq t \leq 4, \\ 0 & \text{otherwise,} \end{cases}$$

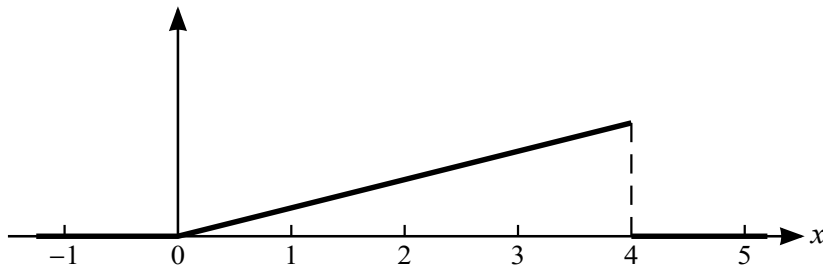
where k is a constant.

(i) Show that $k = \frac{1}{64}$. [3]

(ii) Calculate the probability that two randomly chosen people each spend less than 1 hour on a visit to the museum. [4]

(iii) Find the mean time spent on a visit to the museum. [3]

4



A random variable X takes values between 0 and 4 only and has probability density function as shown in the diagram. Calculate the median of X . [3]

5 The lifetime, X years, of a certain type of battery has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x^2} & 1 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(i) State what the value of a represents in this context. [1]

(ii) Show that $k = \frac{a}{a-1}$. [3]

(iii) Experience has shown that the longest that any battery of this type lasts is 2.5 years. Find the mean lifetime of batteries of this type. [3]

6 The time in minutes taken by people to read a certain booklet is modelled by the random variable T with probability density function given by

$$f(t) = \begin{cases} \frac{1}{2\sqrt{t}} & 4 \leq t \leq 9, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the time within which 90% of people finish reading the booklet. [3]

(ii) Find $E(T)$ and $\text{Var}(T)$. [6]