

Continuous random variables

Question Paper 6

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Continuous random variables
Sub Topic	
Booklet	Question Paper 6

Time Allowed: 59 minutes

Score: /49

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{2}{3}x & 1 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $E(X)$. [3]

(ii) Find $P(X < E(X))$. [2]

(iii) Hence explain whether the mean of X is less than, equal to or greater than the median of X . [2]

2 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x^3} & x \geq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 2$. [2]

(ii) Find $P(1 \leq X \leq 2)$. [2]

(iii) Find $E(X)$. [3]

- 3 The random variable X has probability density function given by

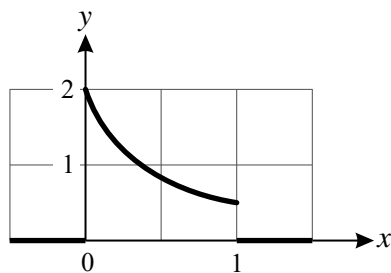
$$f(x) = \begin{cases} \frac{k}{(x+1)^2} & 0 \leq x \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 2$. [2]

(ii) Find a such that $P(X < a) = \frac{1}{5}$. [3]

(iii)



The diagram shows the graph of $y = f(x)$. The median of X is denoted by m . Use the diagram to explain whether $m < 0.5$, $m = 0.5$ or $m > 0.5$. [2]

- 4 At a certain shop the weekly demand, in kilograms, for flour is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} kx^{-\frac{1}{2}} & 4 \leq x \leq 25, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

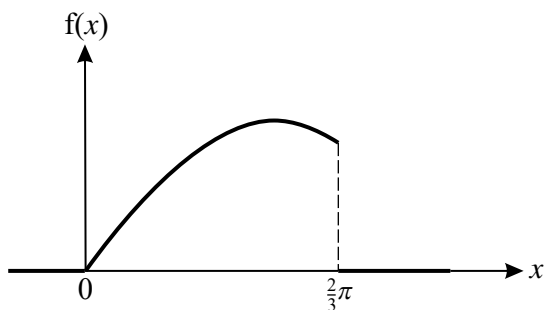
(i) Show that $k = \frac{1}{6}$. [2]

(ii) Calculate the mean weekly demand for flour at the shop. [3]

(iii) At the beginning of one week, the shop has 20 kg of flour in stock. Find the probability that this will not be enough to meet the demand for that week. [2]

(iv) Give a reason why the model may not be realistic. [1]

5



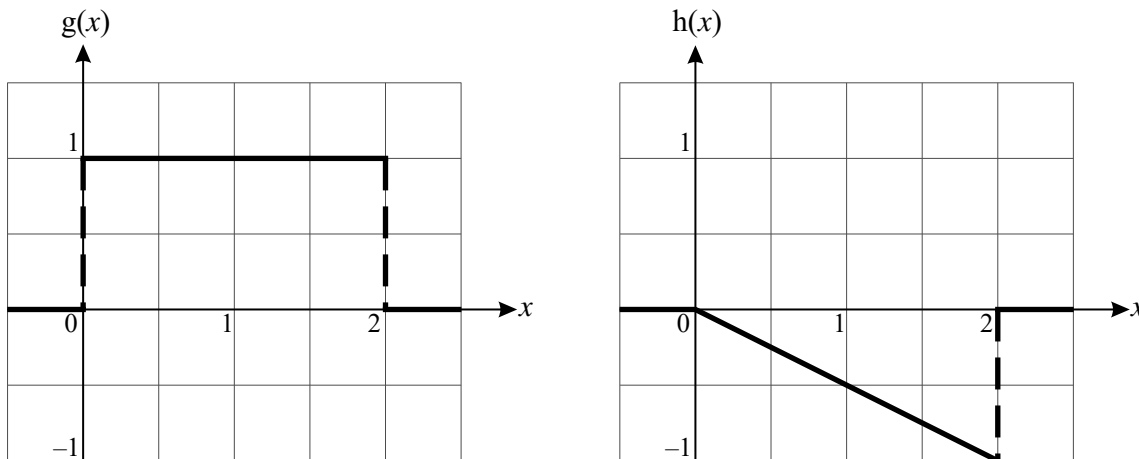
A random variable X has probability density function given by

$$f(x) = \begin{cases} k \sin x & 0 \leq x \leq \frac{2}{3}\pi, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant, as shown in the diagram.

- (i) Show that $k = \frac{2}{3}$. [2]
- (ii) Show that the median of X is 1.32, correct to 3 significant figures. [4]
- (iii) Find $E(X)$. [4]

6 (a)



The diagrams show the graphs of two functions, g and h . For each of the functions g and h , give a reason why it cannot be a probability density function. [2]

(b) The distance, in kilometres, travelled in a given time by a cyclist is represented by the continuous random variable X with probability density function given by

$$f(x) = \begin{cases} \frac{30}{x^2} & 10 \leq x \leq 15, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $E(X) = 30 \ln 1.5$. [3]

(ii) Find the median of X . Find also the probability that X lies between the median and the mean. [5]