Continuous random variables

Question Paper 8

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Continuous random variables
Sub Topic	
Booklet	Question Paper 8

Time Allowed: **77 minutes**

Score: **/64**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The random variable ; has probability density function given b

$$f(x) = \begin{cases} 4x^k & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a positive constant.

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(i) Show that $k = 3$. [2]

(ii) Show that the mean of X is 0.8 and f nd the variance of X. [4]

(iii) Find the upper quartile of X. [2]

(iv) Find the interquartile range of X. [2]

2 At a petrol station cars arrive independently and at random times at constant average rates of 8 cars per hour travelling east and 5 cars per hour travelling west.

(i) Find the probability that, in a quarter-hour period,

(a) one or more cars travelling east and one or more cars travelling west will arrive, [4]

(b) a total of 2 or more cars will arrive. [2]

(ii) Find the approximate probability that, in a 12-hour period, a total of more than 175 cars will arrive. [3]

3 The random variable X denotes the number of hours of cloud cover per day at a weather forecasting centre. The probability density function of X is given by

$$f(x) = \begin{cases} \dfrac{(x-18)^2}{k} & 0 \leqslant x \leqslant 24, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 2016$. [3]

(ii) On how many days in a year of 365 days can the centre expect to have less than 2 hours of cloud cover? [3]

(iii) Find the mean number of hours of cloud cover per day. [4]

4 At a certain airf eld planes land at random times at a constant average rate of one every 10 minutes.

(i) Find the probability that exactly 5 planes will land in a period of one hour. [2]

(ii) Find the probability that at least 2 planes will land in a period of 16 minutes. [3]

(iii) Given that 5 planes landed in an hour, calculate the conditional probability that 1 plane landed in the f rst half hour and 4 in the second half hour. [3]

5 The queuing time, T minutes, for a person queuing at a supermarket checkout has probability density function given by

$$f(t) = \begin{cases} ct(25 - t^2) & 0 \leqslant t \leqslant 5, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

(i) Show that the value of c is $\frac{4}{625}$. [3]

(ii) Find the probability that a person will have to queue for between 2 and 4 minutes. [3]

(iii) Find the mean queuing time. [4]

6 A random variable X has probability density function given by

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $P(X > 1.5)$. [2]

(ii) Find the mean of X. [2]

(iii) Find the median of X. [3]

7 Computer breakdowns occur randomly on average once every 48 hours of use.

(i) Calculate the probability that there will be fewer than 4 breakdowns in 60 hours of use. [3]

(ii) Find the probability that the number of breakdowns in one year (8760 hours) of use is more than 200. [4]

(iii) Independently of the computer breaking down, the computer operator receives phone calls randomly on average twice in every 24-hour period. Find the probability that the total number of phone calls and computer breakdowns in a 60-hour period is exactly 4. [3]