Samples

Question Paper 1

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Sampling and estimation
Sub Topic	Samples
Booklet	Question Paper 1

Time Allowed: **69 minutes**

Score: **/57**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The number of hours that Mrs Hughes spends on her business in a week is normally distributed with mean μ and standard deviation 4.8. In the past the value of μ has been 49.5.

(i) Assuming that μ is still equal to 49.5, fin the probability that in a random sample of 40 weeks the mean time spent on her business in a week is more than 50.3 hours. [4]

Following a change in her arrangements, Mrs Hughes wishes to test whether μ has decreased. She chooses a random sample of 40 weeks and notes that the total number of hours she spent on her business during these weeks is 1920.

(ii) (a) Explain why a one-tail test is appropriate. [1]

(b) Carry out the test at the 6% signif cance level. [4]

(c) Explain whether it was necessary to use the Central Limit theorem in part **(ii) (b)**. [1]

2 The times, in minutes, taken by people to complete a walk are normally distributed with mean μ. The times, t minutes, for a random sample of 80 people were summarised as follows.

$$\Sigma t = 7220 \qquad \Sigma t^2 = 656\,060$$

(i) Calculate a 97% conf dence interval for μ. [6]

(ii) Explain whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

3 Heights of a certain species of animal are known to be normally distributed with standard deviation 0.17 m. A conservationist wishes to obtain a 99% confidenc interval for the population mean, with total width less than 0.2 m. Find the smallest sample size required. [4]

4 **(i)** A random variable X has mean μ and variance σ^2. The mean of a random sample of n values of X is denoted by $\overline{X}$. Give expressions for $\mathrm{E}(\overline{X})$ and $\mathrm{Var}(\overline{X})$. [2]

(ii) The heights, in centimetres, of adult males in Brancot are normally distributed with mean 177.8 and standard deviation 6.1. Find the probability that the mean height of a random sample of 12 adult males from Brancot is less than 176 cm. [3]

(iii) State, with a reason, whether it was necessary to use the Central Limit Theorem in the calculation in part **(ii)**. [1]

5 **(a)** Give a reason why sampling would be required in order to reach a conclusion about

(i) the mean height of adult males in England, [1]

(ii) the mean weight that can be supported by a single cable of a certain type without the cable breaking. [1]

(b) The weights, in kg, of sacks of potatoes are represented by the random variable X with mean μ and standard deviation σ. The weights of a random sample of 500 sacks of potatoes are found and the results are summarised below.

$$n = 500, \qquad \Sigma x = 9850, \qquad \Sigma x^2 = 194\,125.$$

(i) Calculate unbiased estimates of μ and σ^2. [3]

(ii) A further random sample of 60 sacks of potatoes is taken. Using your values from part **(b) (i)**, find the probability that the mean weight of this sample exceeds 19.73 kg. [4]

(iii) Explain whether it was necessary to use the Central Limit Theorem in your calculation in part **(b) (ii)**. [2]

6 The editor of a magazine wishes to obtain the views of a random sample of readers about the future of the magazine.

(i) A sub-editor proposes that they include in one issue of the magazine a questionnaire for readers to complete and return. Give two reasons why the readers who return the questionnaire would not form a random sample. [2]

The editor decides to use a table of random numbers to select a random sample of 50 readers from the 7302 regular readers. These regular readers are numbered from 1 to 7302. The first few random numbers which the editor obtains from the table are as follows.

49757 80239 52038 60882

(ii) Use these random numbers to select the first three members in the sample. [2]

7 There are 18 people in Millie's class. To choose a person at random she numbers the people in the class from 1 to 18 and presses the random number button on her calculator to obtain a 3-digit decimal. Millie then multiplies the first digit in this decimal by two and chooses the person corresponding to this new number. Decimals in which the first digit is zero are ignored.

(i) Give a reason why this is not a satisfactory method of choosing a person. [1]

Millie obtained a random sample of 5 people of her own age by a satisfactory sampling method and found that their heights in metres were 1.66, 1.68, 1.54, 1.65 and 1.57. Heights are known to be normally distributed with variance $0.0052\ \text{m}^2$.

(ii) Find a 98% confidence interval for the mean height of people of Millie's age. [3]

8 Alan wishes to choose one child at random from the eleven children in his music class. The children are numbered 2, 3, 4, and so on, up to 12. Alan then throws two fair dice, each numbered from 1 to 6, and chooses the child whose number is the sum of the scores on the two dice.

(i) Explain why this is an unsatisfactory method of choosing a child. [2]

(ii) Describe brief y a satisfactory method of choosing a child. [2]

9 The daily times, in minutes, that Yu Ming takes showering, getting dressed and having breakfast are independent and have the distributions $N(9, 2.2^2)$, $N(8, 1.3^2)$ and $N(17, 2.6^2)$ respectively. The total daily time that Yu Ming takes for all three activities is denoted by T minutes.

(i) Find the mean and variance of T. [2]

(ii) Yu Ming notes the value of T on each day in a random sample of 70 days and calculates the sample mean. Find the probability that the sample mean is between 33 and 35. [4]