

# Samples

## Question Paper 2

|                   |                         |
|-------------------|-------------------------|
| <b>Level</b>      | International A Level   |
| <b>Subject</b>    | Maths                   |
| <b>Exam Board</b> | CIE                     |
| <b>Topic</b>      | Sampling and estimation |
| <b>Sub Topic</b>  | Samples                 |
| <b>Booklet</b>    | Question Paper 2        |

**Time Allowed:** 62 minutes

**Score:** /51

**Percentage:** /100

**Grade Boundaries:**

| A*   | A      | B   | C     | D     | E   | U    |
|------|--------|-----|-------|-------|-----|------|
| >85% | '77.5% | 70% | 62.5% | 57.5% | 45% | <45% |

1 Jyothi wishes to choose a representative sample of 5 students from the 82 members of her school year.

(i) She considers going into the canteen and choosing a table with five students from her year sitting at it, and using these five people as her sample. Give two reasons why this method is unsatisfactory. [2]

(ii) Jyothi decides to use another method. She numbers all the students in her year from 1 to 82. Then she uses her calculator and generates the following random numbers.

231492      762305      346280

From these numbers, she obtains the student numbers 23, 14, 76, 5, 34 and 62. Explain how Jyothi obtained these student numbers from the list of random numbers. [3]

2 The mean breaking strength of cables made at a certain factory is supposed to be 5 tonnes. The quality control department wishes to test whether the mean breaking strength of cables made by a particular machine is actually less than it should be. They take a random sample of 60 cables. For each cable they find the breaking strength by gradually increasing the tension in the cable and noting the tension when the cable breaks.

(i) Give a reason why it is necessary to take a sample rather than testing all the cables produced by the machine. [1]

(ii) The mean breaking strength of the 60 cables in the sample is found to be 4.95 tonnes. Given that the population standard deviation of breaking strengths is 0.15 tonnes, test at the 1% significance level whether the population mean breaking strength is less than it should be. [4]

(iii) Explain whether it was necessary to use the Central Limit theorem in the solution to part (ii). [2]

3 The score on one throw of a 4-sided die is denoted by the random variable  $X$  with probability distribution as shown in the table.

|            |      |      |      |      |
|------------|------|------|------|------|
| $x$        | 0    | 1    | 2    | 3    |
| $P(X = x)$ | 0.25 | 0.25 | 0.25 | 0.25 |

(i) Show that  $\text{Var}(X) = 1.25$ . [1]

The die is thrown 300 times. The score on each throw is noted and the mean,  $\bar{X}$ , of the 300 scores is found.

(ii) Use a normal distribution to find  $P(\bar{X} < 1.4)$ . [3]

(iii) Justify the use of the normal distribution in part (ii). [1]

4 Marie wants to choose one student at random from Anthea, Bill and Charlie. She throws two fair coins. If both coins show tails she will choose Anthea. If both coins show heads she will choose Bill. If the coins show one of each she will choose Charlie.

(i) Explain why this is not a fair method for choosing the student. [2]

(ii) Describe how Marie could use the two coins to give a fair method for choosing the student. [2]

5 A population has mean 7 and standard deviation 3. A random sample of size  $n$  is chosen from this population.

(i) Write down the mean and standard deviation of the distribution of the sample mean. [2]

(ii) Under what circumstances does the sample mean have

(a) a normal distribution, [1]

(b) an approximately normal distribution? [1]

6 A doctor wishes to investigate the mean fat content in low-fat burgers. He takes a random sample of 15 burgers and sends them to a laboratory where the mass, in grams, of fat in each burger is determined. The results are as follows.

9 7 8 9 6 11 7 9 8 9 8 10 7 9 9

Assume that the mass, in grams, of fat in low-fat burgers is normally distributed with mean  $\mu$  and that the population standard deviation is 1.3.

(i) Calculate a 99% confidence interval for  $\mu$ . [4]

(ii) Explain whether it was necessary to use the Central Limit theorem in the calculation in part (i). [2]

(iii) The manufacturer claims that the mean mass of fat in burgers of this type is 8 g. Use your answer to part (i) to comment on this claim. [2]

7 The lengths of time people take to complete a certain type of puzzle are normally distributed with mean 48.8 minutes and standard deviation 15.6 minutes. The random variable  $X$  represents the time taken in minutes by a randomly chosen person to solve this type of puzzle. The times taken by random samples of 5 people are noted. The mean time  $\bar{X}$  is calculated for each sample.

(i) State the distribution of  $\bar{X}$ , giving the values of any parameters. [2]

(ii) Find  $P(\bar{X} < 50)$ . [3]

8 The lengths of red pencils are normally distributed with mean 6.5 cm and standard deviation 0.23 cm.

(i) Two red pencils are chosen at random. Find the probability that their total length is greater than 12.5 cm. [3]

The lengths of black pencils are normally distributed with mean 11.3 cm and standard deviation 0.46 cm.

(ii) Find the probability that the total length of 3 red pencils is more than 6.7 cm greater than the length of 1 black pencil. [4]

9 Random samples of size 120 are taken from the distribution  $B(15, 0.4)$ .

(i) Describe fully the distribution of the sample mean. [3]

(ii) Find the probability that the mean of a random sample of size 120 is greater than 6.1. [3]