Population

Question Paper 1

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Sampling and estimation
Sub Topic	Population
Booklet	Question Paper 1

Time Allowed: 60 minutes

Score: /50

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 In a survey a random sample of 150 households in Nantville were asked to f ll in a questionnaire about household budgeting.

   (i) The results showed that 33 households owned more than one car. Find an approximate 99% conf dence interval for the proportion of all households in Nantville with more than one car. [4]

   (ii) The results also included the weekly expenditure on food, x dollars, of the households. These were summarised as follows.

   $$n = 150 \qquad \Sigma x = 19\,035 \qquad \Sigma x^2 = 4\,054\,716$$

   Find unbiased estimates of the mean and variance of the weekly expenditure on food of all households in Nantville. [3]

   (iii) The government has a list of all the households in Nantville numbered from 1 to 9526. Describe briefl how to use random numbers to select a sample of 150 households from this list. [3]

2 Following a change in fligh schedules, an airline pilot wished to test whether the mean distance that he flie in a week has changed. He noted the distances, x km, that he fle in 50 randomly chosen weeks and summarised the results as follows.

   $$n = 50 \qquad \Sigma x = 143\,300 \qquad \Sigma x^2 = 410\,900\,000$$

   (i) Calculate unbiased estimates of the population mean and variance. [3]

   (ii) In the past, the mean distance that he fl w in a week was 2850 km. Test, at the 5% signifi ance level, whether the mean distance has changed. [5]

3 The waiting time, T weeks, for a particular operation at a hospital has probability density function given by
   $$f(t) = \begin{cases} \frac{1}{2500}(100t - t^3) & 0 \leqslant t \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

   (i) Given that $E(T) = \frac{16}{3}$, fin Var(T). [3]

   (ii) 10% of patients have to wait more than Q weeks for their operation. Find the value of Q giving your answer correct to the nearest integer. [5]

4 In order to obtain a random sample of people who live in her town, Jane chooses people at random from the telephone directory for her town.

   (i) Give a reason why Jane's method will not give a random sample of people who live in the town.
             [1]

Jane now uses a valid method to choose a random sample of 200 people from her town and f nds that 38 live in apartments.

   (ii) Calculate an approximate 99% conf dence interval for the proportion of all people in Jane's town who live in apartments. [4]

   (iii) Jane uses the same sample to give a conf dence interval of width 0.1 for this proportion. This interval is an x% conf dence interval. Find the value of x. [4]

5 It is claimed that, on average, people following the Losefast diet will lose more than 2 kg per month. The weight losses, x kilograms per month, of a random sample of 200 people following the Losefast diet were recorded and summarised as follows.

$$n = 200 \qquad \Sigma x = 460 \qquad \Sigma x^2 = 1636$$

   (i) Calculate unbiased estimates of the population mean and variance. [3]

   (ii) Test the claim at the 1% signif cance level. [5]

6 The volumes of juice in bottles of Apricola are normally distributed. In a random sample of 8 bottles, the volumes of juice, in millilitres, were found to be as follows.

        332   334   330   328   331   332   329   333

   (i) Find unbiased estimates of the population mean and variance. [3]

A random sample of 50 bottles of Apricola gave unbiased estimates of 331 millilitres and 4.20 millilitres^2 for the population mean and variance respectively.

   (ii) Use this sample of size 50 to calculate a 98% conf dence interval for the population mean. [3]

   (iii) The manufacturer claims that the mean volume of juice in all bottles is 333 millilitres. State, with a reason, whether your answer to part **(ii)** supports this claim. [1]