Population

Question Paper 6

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Sampling and estimation
Sub Topic	Population
Booklet	Question Paper 6

Time Allowed: **80 minutes**

Score: **/66**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The weight, in grams, of a certain type of apple is modelled by the random variable X with mean 62 and standard deviation 8.2. A random sample of 50 apples is selected, and the mean weight in grams, $\overline{X}$, is found.

(i) Describe fully the distribution of $\overline{X}$. [3]

(ii) Find $P(\overline{X} > 64)$. [3]

2 At a power plant, the number of breakdowns per year has a Poisson distribution. In the past the mean number of breakdowns per year has been 4.8. Following some repairs, the management carry out a hypothesis test at the 5% signif cance level to determine whether this mean has decreased. If there is at most 1 breakdown in the following year, they will conclude that the mean has decreased.

(i) State what is meant by a Type I error in this context. [1]

(ii) Find the probability of a Type I error. [2]

(iii) Find the probability of a Type II error if the mean is now 0.9 breakdowns per year. [3]

3 The weights in grams of oranges grown in a certain area are normally distributed with mean μ and standard deviation σ. A random sample of 50 of these oranges was taken, and a 97% conf dence interval for μ based on this sample was (222.1, 232.1).

(i) Calculate unbiased estimates of μ and σ^2. [4]

(ii) Estimate the sample size that would be required in order for a 97% conf dence interval for μ to have width 8. [3]

4 A magazine conducted a survey about the sleeping time of adults. A random sample of 12 adults was chosen from the adults travelling to work on a train.

(i) Give a reason why this is an unsatisfactory sample for the purposes of the survey. [1]

(ii) State a population for which this sample would be satisfactory. [1]

A satisfactory sample of 12 adults gave numbers of hours of sleep as shown below.

4.6 6.8 5.2 6.2 5.7 7.1 6.3 5.6 7.0 5.8 6.5 7.2

(iii) Calculate unbiased estimates of the mean and variance of the sleeping times of adults. [3]

5 The daily takings, $\$x$, for a shop were noted on 30 randomly chosen days. The takings are summarised by $\Sigma x = 31\ 500$, $\Sigma x^2 = 33\ 141\ 816$.

(i) Calculate unbiased estimates of the population mean and variance of the shop's daily takings. [3]

(ii) Calculate a 98% confid nce interval for the mean daily takings. [3]

The mean daily takings for a random sample of n days is found.

(iii) Estimate the value of n for which it is approximately 95% certain that the sample mean does not differ from the population mean by more than \$6. [3]

6 Packets of fis food have weights that are distributed with standard deviation 2.3 g. A random sample of 200 packets is taken. The mean weight of this sample is found to be 99.2 g. Calculate a 99%confidenc interval for the population mean weight. [3]

7 Jenny has to do a statistics project at school on how much pocket money, in dollars, is received by students in her year group. She plans to take a sample of 7 students from her year group, which contains 122 students.

(i) Give a suitable method of taking this sample. [1]

Her sample gives the following results.

13.40 10.60 26.50 20.00 14.50 15.00 16.50

(ii) Find unbiased estimates of the population mean and variance. [3]

(iii) Is the estimated population variance more than, less than or the same as the sample variance? [1]

(iv) Describe what you understand by 'population' in this question. [1]

8 A survey of a random sample of n people found that 61 of them read *The Reporter* newspaper. A symmetric confiden e interval for the true population proportion, p, who read *The Reporter* is $0.1993 < p < 0.2887$.

(i) Find the mid-point of this confid nce interval and use this to f nd the value of n. [3]

(ii) Find the conf dence level of this confid nce interval. [4]

9 Packets of cat food are f lled by a machine.

(i) In a random sample of 10 packets, the weights, in grams, of the packets were as follows.

374.6 377.4 376.1 379.2 371.2 375.0 372.4 378.6 377.1 371.5

Find unbiased estimates of the population mean and variance. [3]

(ii) In a random sample of 200 packets, 38 were found to be underweight. Calculate a 96% confid nce interval for the population proportion of underweight packets. [4]

10 A fair coin is tossed 5 times and the number of heads is recorded.

(i) The random variable X is the number of heads. State the mean and variance of X. [2]

(ii) The number of heads is doubled and denoted by the random variable Y. State the mean and variance of Y. [2]

11 A consumer group, interested in the mean fat content of a particular type of sausage, takes a random sample of 20 sausages and sends them away to be analysed. The percentage of fat in each sausage is as follows.

26 27 28 28 28 29 29 30 30 31 32 32 32 33 33 34 34 34 35 35

Assume that the percentage of fat is normally distributed with mean μ, and that the standard deviation is known to be 3.

(i) Calculate a 98% confidence interval for the population mean percentage of fat. [4]

(ii) The manufacturer claims that the mean percentage of fat in sausages of this type is 30. Use your answer to part **(i)** to determine whether the consumer group should accept this claim. [2]