Hypotesis test

Question Paper 1

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Hypotesis tests
Sub Topic	
Booklet	Question Paper 1

Time Allowed: **64 minutes**

Score: **/53**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The number of accidents on a certain road has a Poisson distribution with mean 3.1 per 12-week period.

(i) Find the probability that there will be exactly 4 accidents during an 18-week period. [3]

Following the building of a new junction on this road, an officer wishes to determine whether the number of accidents per week has decreased. He chooses 15 weeks at random and notes the number of accidents. If there are fewer than 3 accidents altogether he will conclude that the number of accidents per week has decreased. He assumes that a Poisson distribution still applies.

(ii) Find the probability of a Type I error. [3]

(iii) Given that the mean number of accidents per week is now 0.1, find the probability of a Type II error. [3]

(iv) Given that there were 2 accidents during the 15 weeks, explain why it is impossible for the officer to make a Type II error. [1]

2 A researcher wishes to investigate whether the mean height of a certain type of plant in one region is different from the mean height of this type of plant everywhere else. He takes a large random sample of plants from the region and finds the sample mean. He calculates the value of the test statistic, z, and finds that $z = 1.91$.

(i) Explain briefly why the researcher should use a two-tail test. [1]

(ii) Carry out the test at the 4% significance level. [3]

3 It is known that when seeds of a certain type are planted, on average 10% of the resulting plants reach a height of 1 metre. A gardener wishes to investigate whether a new fertiliser will increase this proportion. He plants a random sample of 18 seeds of this type, using the fertiliser, and notes how many of the resulting plants reach a height of 1 metre.

(i) In fact 4 of the 18 plants reach a height of 1 metre. Carry out a hypothesis test at the 8% significance level. [5]

(ii) Explain which of the errors, Type I or Type II, might have been made in part **(i)**. [2]

Later, the gardener plants another random sample of 18 seeds of this type, using the fertiliser, and again carries out a hypothesis test at the 8% significance level.

(iii) Find the probability of a Type I error. [3]

4 At the last election, 70% of people in Apoli supported the president. Luigi believes that the same proportion support the president now. Maria believes that the proportion who support the president now is 35%. In order to test who is right, they agree on a hypothesis test, taking Luigi's belief as the null hypothesis. They will ask 6 people from Apoli, chosen at random, and if more than 3 support the president they will accept Luigi's belief.

(i) Calculate the probability of a Type I error. [3]

(ii) If Maria's belief is true, calculate the probability of a Type II error. [3]

(iii) In fact 2 of the 6 people say that they support the president. State which error, Type I or Type II, might be made. Explain your answer. [2]

5 A traffi office notes the speeds of vehicles as they pass a certain point. In the past the mean of these speeds has been 62.3 km h^{-1} and the standard deviation has been 10.4 km h^{-1}. A speed limit is introduced, and following this, the mean of the speeds of 75 randomly chosen vehicles passing the point is found to be 59.9 km h^{-1}.

(i) Making an assumption that should be stated, test at the 2% significanc level whether the mean speed has decreased since the introduction of the speed limit. [6]

(ii) Explain whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

6 The heights of a certain type of plant have a normal distribution. When the plants are grown without fertilizer, the population mean and standard deviation are 24.0 cm and 4.8 cm respectively. A gardener wishes to test, at the 2% significanc level, whether Hiergro fertilizer will increase the mean height. He treats 150 randomly chosen plants with Hiergro and find that their mean height is 25.0 cm. Assuming that the standard deviation of the heights of plants treated with Hiergro is still 4.8 cm, carry out the test. [5]

7 A cereal manufacturer claims that 25% of cereal packets contain a free gift. Lola suspects that the true proportion is less than 25%. In order to test the manufacturer's claim at the 5% significanc level, she checks a random sample of 20 packets.

(i) Find the critical region for the test. [5]

(ii) Hence f nd the probability of a Type I error. [1]

Lola f nds that 2 packets in her sample contain a free gift.

(iii) State, with a reason, the conclusion she should draw. [2]