Hypotesis test
Question Paper 2

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Hypotesis tests
Sub Topic	
Booklet	Question Paper 2

Time Allowed: 60 minutes

Score: /50

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1   Joshi suspects that a certain die is biased so that the probability of showing a six is less than $\frac{1}{6}$. He plans to throw the die 25 times and if it shows a six on fewer than 2 throws, he will conclude that the die is biased in this way.

  (i) Find the probability of a Type I error and state the signif cance level of the test.   [3]

Joshi now decides to throw the die 100 times. It shows a six on 9 of these throws.

  (ii) Calculate an approximate 95% conf dence interval for the probability of showing a six on one throw of this die.   [4]

2   The management of a factory thinks that the mean time required to complete a particular task is 22 minutes. The times, in minutes, taken by employees to complete this task have a normal distribution with mean μ and standard deviation 3.5. An employee claims that 22 minutes is not long enough for the task. In order to investigate this claim, the times for a random sample of 12 employees are used to test the null hypothesis $\mu = 22$ against the alternative hypothesis $\mu > 22$ at the 5% signif cance level.

  (i) Show that the null hypothesis is rejected in favour of the alternative hypothesis if $\bar{x} > 23.7$ (correct to 3 signif cant f gures), where $\bar{x}$ is the sample mean.   [3]

  (ii) Find the probability of a Type II error given that the actual mean time is 25.8 minutes.   [4]

3   Records show that the distance driven by a bus driver in a week is normally distributed with mean 1150 km and standard deviation 105 km. New driving regulations are introduced and in the next 20 weeks he drives a total of 21 800 km.

  (i) Stating any assumption(s), test, at the 1% signif cance level, whether his mean weekly driving distance has decreased.   [6]

  (ii) A similar test at the 1% signif cance level was carried out using the data from another 20 weeks. State the probability of a Type I error and describe what is meant by a Type I error in this context.
                                                                                                        [2]

4 It is claimed that a certain 6-sided die is biased so that it is more likely to show a six than if it was fair. In order to test this claim at the 10% signif cance level, the die is thrown 10 times and the number of sixes is noted.

   (i) Given that the die shows a six on 3 of the 10 throws, carry out the test. [5]

   On another occasion the same test is carried out again.

   (ii) Find the probability of a Type I error. [3]

   (iii) Explain what is meant by a Type II error in this context. [1]

5 In the past, the number of house sales completed per week by a building company has been modelled by a random variable which has the distribution Po(0.8). Following a publicity campaign, the builders hope that the mean number of sales per week will increase. In order to test at the 5% signif cance level whether this is the case, the total number of sales during the f rst 3 weeks after the campaign is noted. It is assumed that a Poisson model is still appropriate.

   (i) Given that the total number of sales during the 3 weeks is 5, carry out the test. [6]

   (ii) During the following 3 weeks the same test is carried out again, using the same signif cance level. Find the probability of a Type I error. [3]

   (iii) Explain what is meant by a Type I error in this context. [1]

   (iv) State what further information would be required in order to f nd the probability of a Type II error. [1]

6 The number of severe f oods per year in a certain country over the last 100 years has followed a Poisson distribution with mean 1.8. Scientists suspect that global warming has now increased the mean. A hypothesis test, at the 5% signif cance level, is to be carried out to test this suspicion. The number of severe f oods, X, that occur next year will be used for the test.

   (i) Show that the rejection region for the test is $X > 4$. [5]

   (ii) Find the probability of making a Type II error if the mean number of severe f oods is now actually 2.3. [3]