

# Hypotesis test

## Question Paper 3

<b>Level</b>	International A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	CIE
<b>Topic</b>	Hypotesis tests
<b>Sub Topic</b>	
<b>Booklet</b>	Question Paper 3

**Time Allowed:** 64 minutes

**Score:** /53

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

**1** Photographers often need to take many photographs of families until they find a photograph which everyone in the family likes. The number of photographs taken until obtaining one which everybody likes has mean 15.2. A new photographer claims that she can obtain a photograph which everybody likes with fewer photographs taken. To test at the 10% level of significance whether this claim is justified, the numbers of photographs,  $x$ , taken by the new photographer with a random sample of 60 families are recorded. The results are summarised by  $\Sigma x = 890$  and  $\Sigma x^2 = 13\,780$ .

**(i)** Calculate unbiased estimates of the population mean and variance of the number of photographs taken by the new photographer. [3]

**(ii)** State null and alternative hypotheses for the test, and state also the probability that the test results in a Type I error. Say what a Type I error means in the context of the question. [3]

**(iii)** Carry out the test. [4]

**2** It is not known whether a certain coin is fair or biased. In order to perform a hypothesis test, Raj tosses the coin 10 times and counts the number of heads obtained. The probability of obtaining a head on any throw is denoted by  $p$ .

**(i)** The null hypothesis is  $p = 0.5$ . Find the acceptance region for the test, given that the probability of a Type I error is to be at most 0.1. [4]

**(ii)** Calculate the probability of a Type II error in this test if the actual value of  $p$  is 0.7. [3]

**3** The times taken for the pupils in Ming's year group to do their English homework have a normal distribution with standard deviation 15.7 minutes. A teacher estimates that the mean time is 42 minutes. The times taken by a random sample of 3 students from the year group were 27, 35 and 43 minutes. Carry out a hypothesis test at the 10% significance level to determine whether the teacher's estimate for the mean should be accepted, stating the null and alternative hypotheses. [5]

- 4 Every month Susan enters a particular lottery. The lottery company states that the probability,  $p$ , of winning a prize is 0.0017 each month. Susan thinks that the probability of winning is higher than this, and carries out a test based on her 12 lottery results in a one-year period. She accepts the null hypothesis  $p = 0.0017$  if she has no wins in the year and accepts the alternative hypothesis  $p > 0.0017$  if she wins a prize in at least one of the 12 months.
- (i) Find the probability of the test resulting in a Type I error. [2]
  - (ii) If in fact the probability of winning a prize each month is 0.0024, find the probability of the test resulting in a Type II error. [3]
  - (iii) Use a suitable approximation, with  $p = 0.0024$ , to find the probability that in a period of 10 years Susan wins a prize exactly twice. [3]
- 5 Isaac claims that 30% of cars in his town are red. His friend Hardip thinks that the proportion is less than 30%. The boys decided to test Isaac's claim at the 5% significance level and found that 2 cars out of a random sample of 18 were red. Carry out the hypothesis test and state your conclusion. [5]
- 6 In summer the growth rate of grass in a lawn has a normal distribution with mean 3.2 cm per week and standard deviation 1.4 cm per week. A new type of grass is introduced which the manufacturer claims has a slower growth rate. A hypothesis test of this claim at the 5% significance level was carried out using a random sample of 10 lawns that had the new grass. It may be assumed that the growth rate of the new grass has a normal distribution with standard deviation 1.4 cm per week.
- (i) Find the rejection region for the test. [4]
  - (ii) The probability of making a Type II error when the actual value of the mean growth rate of the new grass is  $m$  cm per week is less than 0.5. Use your answer to part (i) to write down an inequality for  $m$ . [1]

- 7 The time taken for Samuel to drive home from work is distributed with mean 46 minutes. Samuel discovers a different route and decides to test at the 5% level whether the mean time has changed. He tries this route on a large number of different days chosen randomly and calculates the mean time.
- (i) State the null and alternative hypotheses for this test. [1]
  - (ii) Samuel calculates the value of his test statistic  $z$  to be  $-1.729$ . What conclusion can he draw? [2]
- 8 Pieces of metal discovered by people using metal detectors are found randomly in fields in a certain area at an average rate of 0.8 pieces per hectare. People using metal detectors in this area have a theory that ploughing the fields increases the average number of pieces of metal found per hectare. After ploughing, they tested this theory and found that a randomly chosen field of area 3 hectares yielded 5 pieces of metal.
- (i) Carry out the test at the 10% level of significance. [6]
  - (ii) What would your conclusion have been if you had tested at the 5% level of significance? [1]
- Jack decides that he will reject the null hypothesis that the average number is 0.8 pieces per hectare if he finds 4 or more pieces of metal in another ploughed field of area 3 hectares.
- (iii) If the true mean after ploughing is 1.4 pieces per hectare, calculate the probability that Jack makes a Type II error. [3]