

# Hypotesis test

## Question Paper 4

<b>Level</b>	International A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	CIE
<b>Topic</b>	Hypotesis tests
<b>Sub Topic</b>	
<b>Booklet</b>	Question Paper 4

**Time Allowed:** 64 minutes

**Score:** /53

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 A manufacturer claims that 20% of sugar-coated chocolate beans are red. George suspects that this percentage is actually less than 20% and so he takes a random sample of 15 chocolate beans and performs a hypothesis test with the null hypothesis  $S = 0.2$  against the alternative hypothesis  $S < 0.2$ . He decides to reject the null hypothesis in favour of the alternative hypothesis if there are 0 or 1 red beans in the sample.
- (i) With reference to this situation, explain what is meant by a Type I error. [1]
- (ii) Find the probability of a Type I error in George's test. [3]
- 2 Flies stick to wet paint at random points. The average number of flies is 2 per square metre. A wall with area  $22 \text{ m}^2$  is painted with a new type of paint which the manufacturer claims is fly-repellent. It is found that 27 flies stick to this wall. Use a suitable approximation to test the manufacturer's claim at the 1% significance level. Take the null hypothesis to be  $\mu = 44$ , where  $\mu$  is the population mean. [5]
- 3 In a research laboratory where plants are studied, the probability of a certain type of plant surviving was 0.35. The laboratory manager changed the growing conditions and wished to test whether the probability of a plant surviving had increased.
- (i) The plants were grown in rows, and when the manager requested a random sample of 8 plants to be taken, the technician took all 8 plants from the front row. Explain what was wrong with the technician's sample. [1]
- (ii) A suitable sample of 8 plants was taken and 4 of these 8 plants survived. State whether the manager's test is one-tailed or two-tailed and also state the null and alternative hypotheses. Using a 5% significance level, find the critical region and carry out the test. [7]
- (iii) State the meaning of a Type II error in the context of the test in part (ii). [1]
- (iv) Find the probability of a Type II error for the test in part (ii) if the probability of a plant surviving is now 0.4. [2]

- 4 The result of a memory test is known to be normally distributed with mean  $\mu$  and standard deviation 1.9. It is required to have a 95% confidence interval for  $\mu$  with a total width of less than 2.0. Find the least possible number of tests needed to achieve this. [4]
- 5 (i) Explain what is meant by
- (a) a Type I error, [1]
  - (b) a Type II error. [1]
- (ii) Roger thinks that a box contains 6 screws and 94 nails. Felix thinks that the box contains 30 screws and 70 nails. In order to test these assumptions they decide to take 5 items at random from the box and inspect them, replacing each item after it has been inspected, and accept Roger's hypothesis (the null hypothesis) if all 5 items are nails.
- (a) Calculate the probability of a Type I error. [4]
  - (b) If Felix's hypothesis (the alternative hypothesis) is true, calculate the probability of a Type II error. [3]
- 6 From previous years' observations, the lengths of salmon in a river were found to be normally distributed with mean 65 cm. A researcher suspects that pollution in water is restricting growth. To test this theory, she measures the length  $x$  cm of a random sample of  $n$  salmon and calculates that  $\bar{x} = 64.3$  and  $s = 4.9$ , where  $s^2$  is the unbiased estimate of the population variance. She then carries out an appropriate hypothesis test.
- (i) Her test statistic  $z$  has a value of  $-1.807$  correct to 3 decimal places. Calculate the value of  $n$ . [3]
  - (ii) Using this test statistic, carry out the hypothesis test at the 5% level of significance and state what her conclusion should be. [4]
- 7 The number of accidents per month at a certain road junction has a Poisson distribution with mean 4.8. A new road sign is introduced warning drivers of the danger ahead, and in a subsequent month 2 accidents occurred.
- (i) A hypothesis test at the 10% level is used to determine whether there were fewer accidents after the new road sign was introduced. Find the critical region for this test and carry out the test. [5]
  - (ii) Find the probability of a Type I error. [2]

- 8 Sami claims that he can read minds. He asks each of 50 people to choose one of the 5 letters A, B, C, D or E. He then tells each person which letter he believes they have chosen. He gets 13 correct. Sami says “This shows that I can read minds, because 13 is more than I would have got right if I were just guessing.”
- (i) State null and alternative hypotheses for a test of Sami’s claim. [1]
- (ii) Test at the 10% significance level whether Sami’s claim is justified. [5]