

# Hypotesis test

## Question Paper 6

<b>Level</b>	International A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	CIE
<b>Topic</b>	Hypotesis tests
<b>Sub Topic</b>	
<b>Booklet</b>	Question Paper 6

**Time Allowed:** 74 minutes

**Score:** /61

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1** A machine is designed to generate random digits between 1 and 5 inclusive. Each digit is supposed to appear with the same probability as the others, but Max claims that the digit 5 is appearing less often than it should. In order to test this claim the manufacturer uses the machine to generate 25 digits and finds that exactly 1 of these digits is a 5.
- (i) Carry out a test of Max's claim at the 2.5% significance level. [5]
  - (ii) Max carried out a similar hypothesis test by generating 1000 digits between 1 and 5 inclusive. The digit 5 appeared 180 times. Without carrying out the test, state the distribution that Max should use, including the values of any parameters. [2]
  - (iii) State what is meant by a Type II error in this context. [1]
- 2** The times taken by students to complete a task are normally distributed with standard deviation 2.4 minutes. A lecturer claims that the mean time is 17.0 minutes. The times taken by a random sample of 5 students were 17.8, 22.4, 16.3, 23.1 and 11.4 minutes. Carry out a hypothesis test at the 5% significance level to determine whether the lecturer's claim should be accepted. [5]
- 3** Leila suspects that a particular six-sided die is biased so that the probability,  $p$ , that it will show a six is greater than  $\frac{1}{6}$ . She tests the die by throwing it 5 times. If it shows a six on 3 or more throws she will conclude that it is biased.
- (i) State what is meant by a Type I error in this situation and calculate the probability of a Type I error. [3]
  - (ii) Assuming that the value of  $p$  is actually  $\frac{2}{3}$ , calculate the probability of a Type II error. [3]
- Leila now throws the die 80 times and it shows a six on 50 throws.
- (iii) Calculate an approximate 96% confidence interval for  $p$ . [4]
- 4** The heights of a certain variety of plant have been found to be normally distributed with mean 75.2 cm and standard deviation 5.7 cm. A biologist suspects that pollution in a certain region is causing the plants to be shorter than usual. He takes a random sample of  $n$  plants of this variety from this region and finds that their mean height is 73.1 cm. He then carries out an appropriate hypothesis test.
- (i) He finds that the value of the test statistic  $z$  is  $-1.563$ , correct to 3 decimal places. Calculate the value of  $n$ . State an assumption necessary for your calculation. [4]
  - (ii) Use this value of the test statistic to carry out the hypothesis test at the 6% significance level. [3]

- 5 The number of cases of asthma per month at a clinic has a Poisson distribution. In the past the mean has been 5.3 cases per month. A new treatment is introduced. In order to test at the 5% significance level whether the mean has decreased, the number of cases in a randomly chosen month is noted.
- (i) Find the critical region for the test and, given that the number of cases is 2, carry out the test. [5]
  - (ii) Explain the meaning of a Type I error in this context and state the probability of a Type I error. [2]
  - (iii) At another clinic the mean number of cases of asthma per month has the independent distribution  $Po(13.1)$ . Assuming that the mean for the first clinic is still 5.3, use a suitable approximating distribution to estimate the probability that the total number of cases in the two clinics in a particular month is more than 20. [5]
- 6 A hockey player found that she scored a goal on 82% of her penalty shots. After attending a coaching course, she scored a goal on 19 out of 20 penalty shots. Making an assumption that should be stated, test at the 10% significance level whether she has improved. [5]
- 7 In the past the weekly profit at a store had mean \$34 600 and standard deviation \$4500. Following a change of ownership, the mean weekly profit for 90 randomly chosen weeks was \$35 400.
- (i) Stating a necessary assumption, test at the 5% significance level whether the mean weekly profit has increased. [6]
  - (ii) State, with a reason, whether it was necessary to use the Central Limit theorem in part (i). [2]
- The mean weekly profit for another random sample of 90 weeks is found and the same test is carried out at the 5% significance level.
- (iii) State the probability of a Type I error. [1]
  - (iv) Given that the population mean weekly profit is now \$36 500, calculate the probability of a Type II error. [5]