Hypotesis test

Question Paper 7

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Hypotesis tests
Sub Topic	
Booklet	Question Paper 7

Time Allowed: **65 minutes**

Score: **/54**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The number of new enquiries per day at an office has a Poisson distribution. In the past the mean has been 3. Following a change of staff, the manager wishes to test, at the 5% significance level, whether the mean has increased.

(i) State the null and alternative hypotheses for this test. [1]

The manager notes the number, N, of new enquiries during a certain 6-day period. She finds that $N = 25$ and then, assuming that the null hypothesis is true, she calculates that $P(N \geqslant 25) = 0.0683$.

(ii) What conclusion should she draw? [2]

2 The weights, X kilograms, of bags of carrots are normally distributed. The mean of X is μ. An inspector wishes to test whether $\mu = 2.0$. He weighs a random sample of 200 bags and his results are summarised as follows.

$$\Sigma x = 430 \qquad \Sigma x^2 = 1290$$

(i) Carry out the test, at the 10% significance level. [6]

(ii) You may now assume that the population variance of X is 1.85. The inspector weighs another random sample of 200 bags and carries out the same test at the 10% significance level.

(a) State the meaning of a Type II error in this context. [1]

(b) Given that $\mu = 2.12$, show that the probability of a Type II error is 0.652, correct to 3 significant figures. [7]

3 **(i)** Deng wishes to test whether a certain coin is biased so that it is more likely to show Heads than Tails. He throws it 12 times. If it shows Heads more than 9 times, he will conclude that the coin is biased. Calculate the significance level of the test. [3]

(ii) Deng throws another coin 100 times in order to test, at the 5% significance level, whether it is biased towards Heads. Find the rejection region for this test. [5]

4 Last year Samir found that the time for his journey to work had mean 45.7 minutes and standard deviation 3.2 minutes. Samir wishes to test whether his journey times have increased this year. He notes the times, in minutes, for a random sample of 8 journeys this year with the following results.

46.2 41.7 49.2 47.1 47.2 48.4 53.7 45.5

It may be assumed that the population of this year's journey times is normally distributed with standard deviation 3.2 minutes.

(i) State, with a reason, whether Samir should use a one-tail or a two-tail test. [2]

(ii) Show that there is no evidence at the 5% signif cance level that Samir's mean journey time has increased. [5]

(iii) State, with a reason, which one of the errors, Type I or Type II, might have been made in carrying out the test in part **(ii)**. [2]

5 Past experience has shown that the heights of a certain variety of rose bush have been normally distributed with mean 85.0 cm. A new fertiliser is used and it is hoped that this will increase the heights. In order to test whether this is the case, a botanist records the heights, x cm, of a large random sample of n rose bushes and calculates that $\bar{x} = 85.7$ and $s = 4.8$, where $\bar{x}$ is the sample mean and s^2 is an unbiased estimate of the population variance. The botanist then carries out an appropriate hypothesis test.

(i) The test statistic, z, has a value of 1.786 correct to 3 decimal places. Calculate the value of n. [3]

(ii) Using this value of the test statistic, carry out the test at the 5% signif cance level. [3]

6 The number of injuries per month at a certain factory has a Poisson distribution. In the past the mean was 2.1 injuries per month. New safety procedures are put in place and the management wishes to use the next 3 months to test, at the 2% signif cance level, whether there are now fewer injuries than before, on average.

(i) Find the critical region for the test. [5]

(ii) Find the probability of a Type I error. [1]

(iii) During the next 3 months there are a total of 3 injuries. Carry out the test. [3]

(iv) Assuming that the mean remains 2.1, calculate an estimate of the probability that there will be fewer than 20 injuries during the next 12 months. [5]