Hypotesis test

Question Paper 9

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Hypotesis tests
Sub Topic	
Booklet	Question Paper 9

Time Allowed: **68 minutes**

Score: **/56**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A machine has produced nails over a long period of time, where the length in millimetres was distributed as N(22.0, 0.19). It is believed that recently the mean length has changed. To test this belief a random sample of 8 nails is taken and the mean length is found to be 21.7 mm. Carry out a hypothesis test at the 5% significanc level to test whether the population mean has changed, assuming that the variance remains the same. [5]

2 At a certain airport 20% of people take longer than an hour to check in. A new computer system is installed, and it is claimed that this will reduce the time to check in. It is decided to accept the claim if, from a random sample of 22 people, the number taking longer than an hour to check in is either 0 or 1.

(i) Calculate the signifi ance level of the test. [3]

(ii) State the probability that a Type I error occurs. [1]

(iii) Calculate the probability that a Type II error occurs if the probability that a person takes longer than an hour to check in is now 0.09. [3]

3 The number of cars caught speeding on a certain length of motorway is 7.2 per day, on average. Speed cameras are introduced and the results shown in the following table are those from a random selection of 40 days after this.

Number of cars caught speeding	4	5	6	7	8	9	10
Number of days	5	7	8	10	5	2	3

(i) Calculate unbiased estimates of the population mean and variance of the number of cars per day caught speeding after the speed cameras were introduced. [3]

(ii) Taking the null hypothesis H_0 to be $\mu = 7.2$, test at the 5% level whether there is evidence that the introduction of speed cameras has resulted in a reduction in the number of cars caught speeding. [5]

(iii) State what is meant by a Type I error in words relating to the context of the test in part **(ii)**. Without further calculation, illustrate on a suitable diagram the region representing the probability of this Type I error. [3]

4 A study of a large sample of books by a particular author shows that the number of words per sentence can be modelled by a normal distribution with mean 21.2 and standard deviation 7.3. A researcher claims to have discovered a previously unknown book by this author. The mean length of 90 sentences chosen at random in this book is found to be 19.4 words.

(i) Assuming the population standard deviation of sentence lengths in this book is also 7.3, test at the 5% level of significanc whether the mean sentence length is the same as the author's. State your null and alternative hypotheses. [5]

(ii) State in words relating to the context of the test what is meant by a Type I error and state the probability of a Type I error in the test in part **(i)**. [2]

5 Each multiple choice question in a test has 4 suggested answers, exactly one of which is correct. Rehka knows nothing about the subject of the test, but claims that she has a special method for answering the questions that is better than just guessing. There are 60 questions in the test, and Rehka gets 22 correct.

(i) State null and alternative hypotheses for a test of Rehka's claim. [1]

(ii) Using a normal approximation, test at the 5% signific nce level whether Rehka's claim is justifi d. [4]

6 The lectures in a mathematics department are scheduled to last 54 minutes, and the times of individual lectures may be assumed to have a normal distribution with mean μ minutes and standard deviation 3.1 minutes. One of the students commented that, on average, the lectures seemed too short. To investigate this, the times for a random sample of 10 lectures were used to test the null hypothesis

$\mu = 54$ against the alternative hypothesis $\mu < 54$ at the 10% signific nce level.

(i) Show that the null hypothesis is rejected in favour of the alternative hypothesis if $\bar{x} < 52.74$, where $\bar{x}$ minutes is the sample mean. [4]

(ii) Find the probability of a Type II error given that the actual mean length of lectures is 51.5 minutes. [4]

7 Before attending a basketball course, a player found that 60% of his shots made a score. After attending the course the player claimed he had improved. In his next game he tried 12 shots and scored in 10 of them. Assuming shots to be independent, test this claim at the 10% significance level.[5]

8 Over a long period of time it is found that the time spent at cash withdrawal points follows a normal distribution with mean 2.1 minutes and standard deviation 0.9 minutes. A new system is tried out, to speed up the procedure. The null hypothesis is that the mean time spent is the same under the new system as previously. It is decided to reject the null hypothesis and accept that the new system is quicker if the mean withdrawal time from a random sample of 20 cash withdrawals is less than 1.7 minutes. Assume that, for the new system, the standard deviation is still 0.9 minutes, and the time spent still follows a normal distribution.

(i) Calculate the probability of a Type I error. [4]

(ii) If the mean withdrawal time under the new system is actually 1.5 minutes, calculate the probability of a Type II error. [4]