Differentiation - Logs, Exponentials, Rates of Change

Question Paper 3

Level	A Level
Subject	Maths
Exam Board	OCR
Module	Core 3
Topic	Differentiation
Sub Topic	Differentiation - Logs, Exponentials, Rates of Change
Booklet	Question Paper - 3

Time Allowed: **61 minutes**

Score: **/51**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 A giant spherical balloon is being inflated in a theme park. The radius of the balloon is increasing at a rate of 12 cm per hour. Find the rate at which the surface area of the balloon is increasing at the instant when the radius is 150 cm. Give your answer in cm^2 per hour correct to 2 significant figures.

[Surface area of sphere = $4\pi r^2$.] **[3]**

2 **(a)** Leaking oil is forming a circular patch on the surface of the sea. The area of the patch is increasing at a rate of 250 square metres per hour. Find the rate at which the radius of the patch is increasing at the instant when the area of the patch is 1900 square metres. Give your answer correct to 2 significant figures. **[4]**

(b) The mass of a substance is decreasing exponentially. Its mass now is 150 grams and its mass, m grams, at a time t years from now is given by

$$m = 150e^{-kt},$$

where k is a positive constant. Find, in terms of k, the number of years from now at which the mass will be decreasing at a rate of 3 grams per year. **[3]**

3 The mass, M grams, of a certain substance is increasing exponentially so that, at time t hours, the mass is given by

$$M = 40e^{kt},$$

where k is a constant. The following table shows certain values of t and M.

t	0	21	63
M		80	

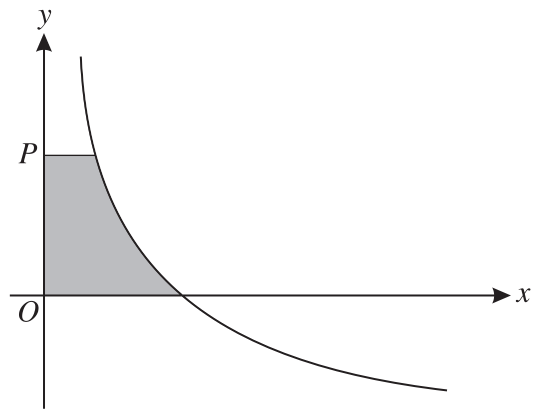
(i) In either order,

(a) find the values missing from the table, **[3]**

(b) determine the value of k. **[2]**

(ii) Find the rate at which the mass is increasing when $t = 21$. **[3]**

4



The diagram shows the curve with equation

$$y = \frac{6}{\sqrt{x}} - 3.$$

The point P has coordinates $(0, p)$. The shaded region is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The shaded region is rotated completely about the y-axis to form a solid of volume V.

(i) Show that $V = 16\pi\left(1 - \frac{27}{(p+3)^3}\right)$. **[6]**

(ii) It is given that P is moving along the y-axis in such a way that, at time t, the variables p and t are related by

$$\frac{dp}{dt} = \tfrac{1}{3}p + 1.$$

Find the value of $\frac{dV}{dt}$ at the instant when $p = 9$. **[4]**

5 Earth is being added to a pile so that, when the height of the pile is h metres, its volume is V cubic metres, where

$$V = (h^6 + 16)^{\frac{1}{2}} - 4.$$

(i) Find the value of $\frac{dV}{dh}$ when $h = 2$. [3]

(ii) The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate, in metres per hour, at which the height of the pile is increasing at the instant when $h = 2$. Give your answer correct to 2 significant figures. [3]

6 A curve has equation $y = \dfrac{x\mathrm{e}^{2x}}{x+k}$, where k is a non-zero constant.

(i) Differentiate $x\mathrm{e}^{2x}$, and show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{e}^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$. [5]

(ii) Given that the curve has exactly one stationary point, find the value of k, and determine the exact coordinates of the stationary point. [5]

7 **(i)** Given that $x = (4t+9)^{\frac{1}{2}}$ and $y = 6\mathrm{e}^{\frac{1}{2}x+1}$, find expressions for $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [4]

(ii) Hence find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ when $t = 4$, giving your answer correct to 3 significant figures. [3]