# Vectors

## Question Paper 4

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|---|---|
| **Level** | International A Level |
| **Subject** | Maths |
| **Exam Board** | CIE |
| **Topic** | Vectors |
| **Sub Topic** | |
| **Booklet** | Question Paper 4 |

**Time Allowed:** **58 minutes**

**Score:** **/48**

**Percentage:** **/100**

**Grade Boundaries:**

| A* | A | B | C | D | E | U |
|---|---|---|---|---|---|---|
| >85% | '77.5% | 70% | 62.5% | 57.5% | 45% | <45% |

**1** Relative to an origin $O$, the position vectors of the points $A$, $B$ and $C$ are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}.$$

Find

**(i)** the unit vector in the direction of $\overrightarrow{AB}$, [3]

**(ii)** the value of the constant $p$ for which angle $BOC = 90°$. [2]

**2** Relative to an origin $O$, the point $A$ has position vector $4\mathbf{i} + 7\mathbf{j} - p\mathbf{k}$ and the point $B$ has position vector $8\mathbf{i} - \mathbf{j} - p\mathbf{k}$, where $p$ is a constant.

**(i)** Find $\overrightarrow{OA}.\overrightarrow{OB}$. [2]

**(ii)** Hence show that there are no real values of $p$ for which $OA$ and $OB$ are perpendicular to each other. [1]

**(iii)** Find the values of $p$ for which angle $AOB = 60°$. [4]

**3** Relative to an origin $O$, the position vectors of points $A$ and $B$ are given by

$$\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k},$$

where $p$ is a constant.

**(i)** Find the value of $p$ for which angle $AOB$ is $90°$. [3]

**(ii)** In the case where $p = 4$, find the vector which has magnitude 28 and is in the same direction as $\overrightarrow{AB}$. [4]

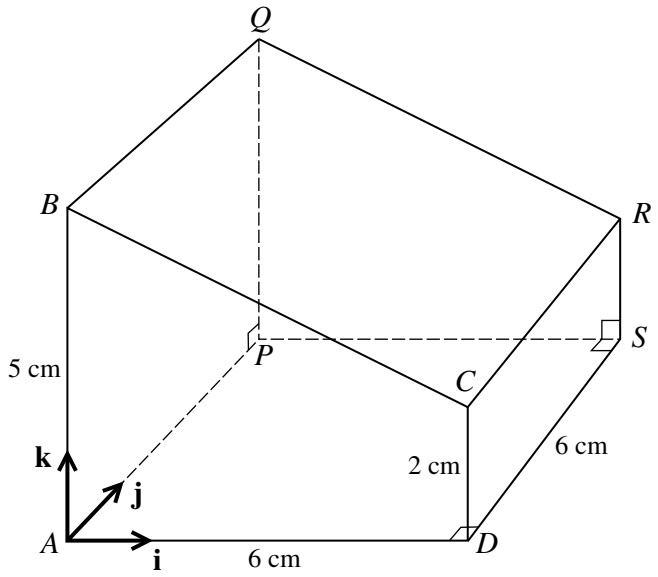
**4** Relative to an origin $O$, the position vectors of points $A$ and $B$ are $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ respectively.

**(i)** Use a scalar product to find angle $BOA$. [4]

The point $C$ is the mid-point of $AB$. The point $D$ is such that $\overrightarrow{OD} = 2\overrightarrow{OB}$.

**(ii)** Find $\overrightarrow{DC}$. [4]

**5**




The diagram shows a prism $ABCDPQRS$ with a horizontal square base $APSD$ with sides of length 6 cm. The cross-section $ABCD$ is a trapezium and is such that the vertical edges $AB$ and $DC$ are of lengths 5 cm and 2 cm respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $AD$, $AP$ and $AB$ respectively.

**(i)** Express each of the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

**(ii)** Use a scalar product to calculate angle $PCQ$. [4]

**6** Relative to the origin $O$, the position vectors of the points $A$, $B$ and $C$ are given by

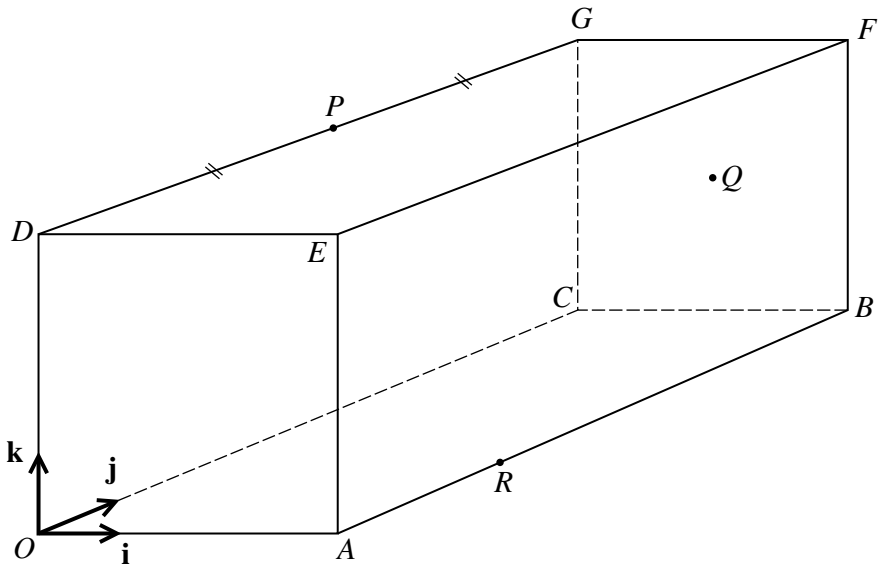
$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 10 \\ 0 \\ 6 \end{pmatrix}.$$

**(i)** Find angle $ABC$. [6]

The point $D$ is such that $ABCD$ is a parallelogram.

**(ii)** Find the position vector of $D$. [2]

**7**




In the diagram, $OABCDEFG$ is a rectangular block in which $OA = OD = 6$ cm and $AB = 12$ cm. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point $P$ is the mid-point of $DG$, $Q$ is the centre of the square face $CBFG$ and $R$ lies on $AB$ such that $AR = 4$ cm.

**(i)** Express each of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ in terms of **i**, **j** and **k**. [3]

**(ii)** Use a scalar product to find angle $RQP$. [4]