

Discrete Random Variables

Question Paper 4

| | |
|-------------------|---------------------------|
| Level | A Level |
| Subject | Maths |
| Exam Board | OCR - MEI |
| Module | Statistics 1 |
| Topic | Discrete Random Variables |
| Sub Topic | Discrete Random Variables |
| Booklet | Question Paper 4 |

Time Allowed: 58 minutes

Score: /48

Percentage: /100

Grade Boundaries:

| A* | A | B | C | D | E | U |
|------|-------|-----|-------|-------|-----|------|
| >85% | 77.5% | 70% | 62.5% | 57.5% | 45% | <45% |

- 1 The maximum temperatures x degrees Celsius recorded during each month of 2005 in Cambridge are given in the table below.

| Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|-----|-----|------|------|------|------|------|------|------|------|------|-----|
| 9.2 | 7.1 | 10.7 | 14.2 | 16.6 | 21.8 | 22.0 | 22.6 | 21.1 | 17.4 | 10.1 | 7.8 |

These data are summarised by $n = 12$, $\Sigma x = 180.6$, $\Sigma x^2 = 3107.56$.

- (i) Calculate the mean and standard deviation of the data. [3]
- (ii) Determine whether there are any outliers. [3]
- (iii) The formula $y = 1.8x + 32$ is used to convert degrees Celsius to degrees Fahrenheit. Find the mean and standard deviation of the 2005 maximum temperatures in degrees Fahrenheit. [3]
- (iv) In New York, the monthly maximum temperatures are recorded in degrees Fahrenheit. In 2005 the mean was 63.7 and the standard deviation was 16.0. Briefly compare the maximum monthly temperatures in Cambridge and New York in 2005. [2]

The total numbers of hours of sunshine recorded in Cambridge during the month of January for each of the last 48 years are summarised below.

| Hours h | $70 \leq h < 100$ | $100 \leq h < 110$ | $110 \leq h < 120$ | $120 \leq h < 150$ | $150 \leq h < 170$ | $170 \leq h < 190$ |
|-----------------|-------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Number of years | 6 | 8 | 10 | 11 | 10 | 3 |

- (v) Draw a cumulative frequency graph for these data. [5]
- (vi) Use your graph to estimate the 90th percentile. [2]
- 2 The marks x scored by a sample of 56 students in an examination are summarised by

$$n = 56, \quad \Sigma x = 3026, \quad \Sigma x^2 = 178\,890.$$

- (i) Calculate the mean and standard deviation of the marks. [3]
- (ii) The highest mark scored by any of the 56 students in the examination was 93. Show that this result may be considered to be an outlier. [2]
- (iii) The formula $y = 1.2x - 10$ is used to scale the marks. Find the mean and standard deviation of the scaled marks. [3]

- 3 In a phone-in competition run by a local radio station, listeners are given the names of 7 local personalities and are told that 4 of them are in the studio. Competitors phone in and guess which 4 are in the studio.

(i) Show that the probability that a randomly selected competitor guesses all 4 correctly is $\frac{1}{35}$. [2]

Let X represent the number of correct guesses made by a randomly selected competitor. The probability distribution of X is shown in the table.

| | | | | | |
|------------|---|----------------|-----------------|-----------------|----------------|
| r | 0 | 1 | 2 | 3 | 4 |
| $P(X = r)$ | 0 | $\frac{4}{35}$ | $\frac{18}{35}$ | $\frac{12}{35}$ | $\frac{1}{35}$ |

(ii) Find the expectation and variance of X . [5]

- 4 A fair six-sided die is rolled twice. The random variable X represents the higher of the two scores. The probability distribution of X is given by the formula

$$P(X = r) = k(2r - 1) \text{ for } r = 1, 2, 3, 4, 5, 6.$$

(i) Copy and complete the following probability table and hence find the exact value of k , giving your answer as a fraction in its simplest form. [3]

| | | | | | | |
|------------|-----|---|---|---|---|-------|
| r | 1 | 2 | 3 | 4 | 5 | 6 |
| $P(X = r)$ | k | | | | | $11k$ |

(ii) Find the mean of X . [2]

A fair six-sided die is rolled three times.

(iii) Find the probability that the total score is 16. [3]

- 5** The score, X , obtained on a given throw of a biased, four-faced die is given by the probability distribution

$$P(X = r) = kr(8 - r) \text{ for } r = 1, 2, 3, 4.$$

- (i)** Show that $k = \frac{1}{50}$. [2]
- (ii)** Calculate $E(X)$ and $\text{Var}(X)$. [5]