

Logarithms & Exponentials

Question Paper 4

Level	A Level
Subject	Maths
Exam Board	OCR
Module	Core 2
Topic	Algebra
Sub Topic	Logarithms & Exponentials
Booklet	Question Paper - 4

Time Allowed: 57 minutes

Score: /47

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

- 1 (a) (i) Express $\log_3(4x + 7) - \log_3 x$ as a single logarithm. [1]
(ii) Hence solve the equation $\log_3(4x + 7) - \log_3 x = 2$. [3]

(b) Use the trapezium rule, with two strips of width 3, to find an approximate value for

$$\int_3^9 \log_{10} x \, dx,$$

giving your answer correct to 3 significant figures. [4]

- 2 On its first trip between Malby and Grenlish, a steam train uses 1.5 tonnes of coal. As the train does more trips, it becomes less efficient so that each subsequent trip uses 2% more coal than the previous trip.

(i) Show that the amount of coal used on the fifth trip is 1.624 tonnes, correct to 4 significant figures. [2]

(ii) There are 39 tonnes of coal available. An engineer wishes to calculate N , the total number of trips possible. Show that N satisfies the inequality

$$1.02^N \leq 1.52. \quad [4]$$

(iii) Hence, by using logarithms, find the greatest number of trips possible. [4]

- 3 (i) Express each of the following in terms of $\log_{10} x$ and $\log_{10} y$.

(a) $\log_{10}\left(\frac{x}{y}\right)$ [1]

(b) $\log_{10}(10x^2y)$ [3]

(ii) Given that

$$2 \log_{10}\left(\frac{x}{y}\right) = 1 + \log_{10}(10x^2y),$$

find the value of y correct to 3 decimal places. [4]

- 4 (i) On a single diagram, sketch the curves with the following equations. In each case state the coordinates of any points of intersection with the axes.

(a) $y = a^x$, where a is a constant such that $a > 1$. [2]

(b) $y = 2b^x$, where b is a constant such that $0 < b < 1$. [2]

- (ii) The curves in part (i) intersect at the point P . Prove that the x -coordinate of P is

$$\frac{1}{\log_2 a - \log_2 b}. \quad [5]$$

- 5 (a) An arithmetic progression has first term $\log_2 27$ and common difference $\log_2 x$.

(i) Show that the fourth term can be written as $\log_2 (27x^3)$. [3]

(ii) Given that the fourth term is 6, find the exact value of x . [2]

- (b) A geometric progression has first term $\log_2 27$ and common ratio $\log_2 y$.

(i) Find the set of values of y for which the geometric progression has a sum to infinity. [2]

(ii) Find the exact value of y for which the sum to infinity of the geometric progression is 3. [5]