# Completing the square

## Question Paper 1

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|---|---|
| Level | A Level |
| Subject | Mathematics (Pure) |
| Exam Board | AQA |
| Module | Core 1 |
| Topic | Algebra |
| Sub Topic | Completing the square |
| Booklet | Question Paper 1 |

**Time Allowed:** **79 minutes**

**Score:** **/66**

**Percentage:** **/100**

**Grade Boundaries:**

| A* | A | B | C | D | E | U |
|---|---|---|---|---|---|---|
| >85% | '77.5% | 70% | 62.5% | 57.5% | 45% | <45% |

**1** (a) (i) Express $x^2 - 6x + 11$ in the form $(x - p)^2 + q$.

**(2)**

(ii) **Use the result from part (a)(i)** to show that the equation $x^2 - 6x + 11 = 0$ has no real solutions.

**(2)**

(b) A curve has equation $y = x^2 - 6x + 11$.

(i) Find the coordinates of the vertex of the curve.

**(2)**

(ii) Sketch the curve, indicating the value of $y$ where the curve crosses the $y$-axis.

**(3)**

(iii) Describe the geometrical transformation that maps the curve with equation $y = x^2 - 6x + 11$ onto the curve with equation $y = x^2$.

**(3)**

**(Total 12 marks)**

**2** (a) (i) Express $2x^2 + 6x + 5$ in the form $2(x + p)^2 + q$, where $p$ and $q$ are rational numbers.

**(2)**

(ii) Hence write down the minimum value of $2x^2 + 6x + 5$.

**(1)**

(b) The point $A$ has coordinates $(-3, 5)$ and the point $B$ has coordinates $(x, 3x + 9)$.

(i) Show that $AB^2 = 5(2x^2 + 6x + 5)$.

**(3)**

(ii) Use your result from part (a)(ii) to find the minimum value of the length $AB$ as $x$ varies, giving your answer in the form $\frac{1}{2}\sqrt{n}$, where $n$ is an integer.

**(2)**

**(Total 8 marks)**

**3** (a) Factorise $x^2 - 4x - 12$.

**(1)**

(b) Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes.

**(4)**

(c) (i) Express $x^2 - 4x - 12$ in the form $(x - p)^2 - q$, where $p$ and $q$ are positive integers.

**(2)**

(ii) Hence find the minimum value of $x^2 - 4x - 12$.

**(1)**

(d) The curve with equation $y = x^2 - 4x - 12$ is translated by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

Find an equation of the new curve. You need not simplify your answer.

**(2)**

**(Total 10 marks)**

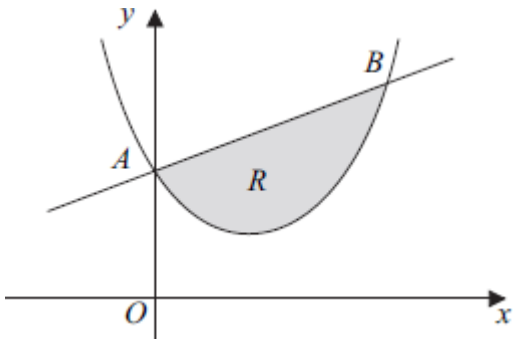
**4** (a) (i) Express $x^2 - 3x + 5$ in the form $(x - p)^2 + q$.

**(2)**

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = x^2 - 3x + 5$.

**(1)**

(b) The curve $C$ with equation $y = x^2 - 3x + 5$ and the straight line $y = x + 5$ intersect at the point $A(0, 5)$ and at the point $B$, as shown in the diagram below.




(i) Find the coordinates of the point $B$.

**(3)**

(ii) Find $\int (x^2 - 3x + 5)\, dx$.

**(3)**

(iii) Find the area of the shaded region $R$ bounded by the curve $C$ and the line segment $AB$.

**(4)**

**(Total 13 marks)**

**5** (a) (i) Express $4 - 10x - x^2$ in the form $p - (x + q)^2$

**(2)**

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = 4 - 10x - x^2$.

**(1)**

(b) The curve $C$ has equation $y = 4 - 10x - x^2$ and the line $L$ has equation $y = k(4x - 13)$, where $k$ is a constant.

(i) Show that the $x$-coordinates of any points of intersection of the curve $C$ with the line $L$ satisfy the equation

$$x^2 + 2(2k + 5)x - (13k + 4) = 0$$

**(1)**

(ii) Given that the curve $C$ and the line $L$ intersect in two distinct points, show that

$$4k^2 + 33k + 29 > 0$$

**(3)**

(iii) Solve the inequality $4k^2 + 33k + 29 > 0$.

**(4)**

**(Total 11 marks)**

**6** (a) Express $x^2 + 5x + 7$ in the form $(x + p)^2 + q$, where $p$ and $q$ are rational numbers.

**(3)**

(b) A curve has equation $y = x^2 + 5x + 7$.

(i) Find the coordinates of the vertex of the curve.

**(2)**

(ii) State the equation of the line of symmetry of the curve.

**(1)**

(iii) Sketch the curve, stating the value of the intercept on the $y$-axis.

**(3)**

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 5x + 7$.

**(3)**

**(Total 12 marks)**