

# Completing the square

## Question Paper 3

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 1
Topic	Algebra
Sub Topic	Completing the square
Booklet	Question Paper 3

**Time Allowed:** 87 minutes

**Score:** /72

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1**
- (a) Factorise  $9 - 8x - x^2$ . (2)
- (b) Show that  $25 - (x + 4)^2$  can be written as  $9 - 8x - x^2$ . (1)
- (c) A curve has equation  $y = 9 - 8x - x^2$ .
- (i) Write down the equation of its line of symmetry. (1)
- (ii) Find the coordinates of its vertex. (2)
- (iii) Sketch the curve, indicating the values of the intercepts on the  $x$ -axis and the  $y$ -axis. (3)
- (Total 9 marks)**

- 2**
- (a) Express  $x^2 - 3x + 4$  in the form  $(x - p)^2 + q$ , where  $p$  and  $q$  are rational numbers. (2)
- (b) Hence write down the minimum value of the expression  $x^2 - 3x + 4$ . (1)
- (c) Describe the geometrical transformation that maps the graph of  $y = x^2$  onto the graph of  $y = x^2 - 3x + 4$ . (3)
- (Total 6 marks)**

- 3**
- (a) (i) Express  $x^2 + 10x + 19$  in the form  $(x + p)^2 + q$ , where  $p$  and  $q$  are integers. (2)
- (ii) Write down the coordinates of the vertex (minimum point) of the curve with equation  $y = x^2 + 10x + 19$ . (2)
- (iii) Write down the equation of the line of symmetry of the curve  $y = x^2 + 10x + 19$ . (1)

(iv) Describe geometrically the transformation that maps the graph of  $y = x^2$  onto the graph of  $y = x^2 + 10x + 19$ .

(3)

(b) Determine the coordinates of the points of intersection of the line  $y = x + 11$  and the curve  $y = x^2 + 10x + 19$ .

(4)

(Total 12 marks)

4

(a) (i) Express  $x^2 - 4x + 9$  in the form  $(x - p)^2 + q$ , where  $p$  and  $q$  are integers.

(2)

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation  $y = x^2 - 4x + 9$ .

(2)

(b) The line  $L$  has equation  $y + 2x = 12$  and the curve  $C$  has equation  $y = x^2 - 4x + 9$ .

(i) Show that the  $x$ -coordinates of the points of intersection of  $L$  and  $C$  satisfy the equation

$$x^2 - 2x - 3 = 0$$

(1)

(ii) Hence find the coordinates of the points of intersection of  $L$  and  $C$ .

(4)

(Total 9 marks)

5

A circle with centre  $C$  has equation  $x^2 + y^2 - 8x + 6y = 11$ .

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

(3)

(b) Write down:

(i) the coordinates of  $C$ ;

(1)

(ii) the radius of the circle.

(1)

(c) The point  $O$  has coordinates  $(0, 0)$ .

(i) Find the length of  $CO$ .

(2)

(ii) Hence determine whether the point  $O$  lies inside or outside the circle, giving a reason for your answer.

(2)

(Total 9 marks)

6

(a) Express  $x^2 + 8x + 19$  in the form  $(x + p)^2 + q$ , where  $p$  and  $q$  are integers.

(2)

(b) Hence, or otherwise, show that the equation  $x^2 + 8x + 19 = 0$  has no real solutions.

(2)

(c) Sketch the graph of  $y = x^2 + 8x + 19$ , stating the coordinates of the minimum point and the point where the graph crosses the  $y$ -axis.

(3)

(d) Describe geometrically the transformation that maps the graph of  $y = x^2$  onto the graph of  $y = x^2 + 8x + 19$ .

(3)

(Total 10 marks)

7

A circle has equation  $x^2 + y^2 - 4x - 14 = 0$ .

(a) Find:

(i) the coordinates of the centre of the circle;

(3)

(ii) the radius of the circle in the form  $p\sqrt{2}$ , where  $p$  is an integer.

(3)

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord.

**(3)**

(c) A line has equation  $y = 2k - x$ , where  $k$  is a constant.

(i) Show that the  $x$ -coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k + 1)x + 2k^2 - 7 = 0$$

**(3)**

(ii) Find the values of  $k$  for which the equation

$$x^2 - 2(k + 1)x + 2k^2 - 7 = 0$$

has equal roots.

**(4)**

(iii) Describe the geometrical relationship between the line and the circle when  $k$  takes either of the values found in part (c)(ii).

**(1)**

**(Total 17 marks)**