

Differentiation

Question Paper 3

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 2
Topic	Calculus
Sub Topic	Differentiation
Booklet	Question Paper 3

Time Allowed: 85 minutes

Score: /71

Percentage: /100

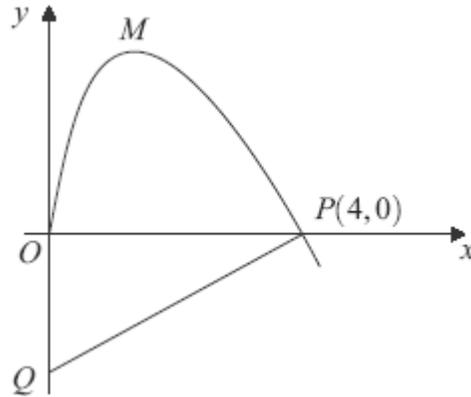
Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1

A curve, drawn from the origin O , crosses the x -axis at the point $P(4, 0)$.

The normal to the curve at P meets the y -axis at the point Q , as shown in the diagram.



The curve, defined for $x \geq 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

- (a) (i) Find $\frac{dy}{dx}$. (3)
- (ii) Show that the gradient of the curve at $P(4, 0)$ is -2 . (2)
- (iii) Find an equation of the normal to the curve at $P(4, 0)$. (3)
- (iv) Find the y -coordinate of Q and hence find the area of triangle OPQ . (3)
- (v) The curve has a maximum point M . Find the x -coordinate of M . (3)
- (b) (i) Find $\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}} \right) dx$. (3)
- (ii) Find the total area of the region bounded by the curve and the lines PQ and QO . (3)
- (Total 20 marks)**

2

- (a) Write $\sqrt{x^3}$ in the form x^k , where k is a fraction.

(1)

(b) A curve, defined for $x \geq 0$, has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find $\frac{dy}{dx}$.

(3)

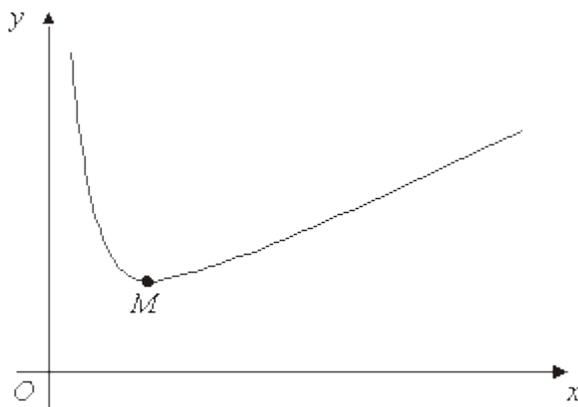
(ii) Find the equation of the tangent to the curve at the point where $x = 4$, giving your answer in the form $y = mx + c$.

(5)

(Total 9 marks)

3

A curve C is defined for $x > 0$ by the equation $y = x + 1 + \frac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \frac{4}{x^2}$, find $\frac{dy}{dx}$.

(3)

(ii) The curve C has a minimum point M . Find the coordinates of M .

(4)

(iii) Find an equation of the normal to C at the point $(1, 6)$.

(4)

(b) (i) Find $\int \left(x + 1 + \frac{4}{x^2} \right) dx$. (3)

(ii) Hence find the area of the region bounded by the curve C , the lines $x = 1$ and $x = 4$ and the x -axis.

(2)
(Total 16 marks)

4 A curve is defined for $x > 0$ by the equation

$$y = \left(1 + \frac{2}{x} \right)^2$$

The point P lies on the curve where $x = 2$.

(a) Find the y -coordinate of P . (1)

(b) Expand $\left(1 + \frac{2}{x} \right)^2$. (2)

(c) Find $\frac{dy}{dx}$. (3)

(d) Hence show that the gradient of the curve at P is -2 . (2)

(e) Find the equation of the normal to the curve at P , giving your answer in the form $x + by + c = 0$, where b and c are integers. (4)
(Total 12 marks)

5 Given that $y = 16x + x^{-1}$, find the two values of x for which $\frac{dy}{dx} = 0$.

(Total 5 marks)

- 6** (a) Use logarithms to solve the equation $0.8^x = 0.05$, giving your answer to three decimal places. **(3)**
- (b) An infinite geometric series has common ratio r . The sum to infinity of the series is five times the first term of the series.
- (i) Show that $r = 0.8$. **(3)**
- (ii) Given that the first term of the series is 20, find the least value of n such that the n th term of the series is less than 1. **(3)**

(Total 9 marks)