Differentiation

Question Paper 3

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 1
Topic	Calculus
Sub Topic	Differentiation
Booklet	Question Paper 3

Time Allowed: **85 minutes**

Score: **/70**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

$$x = \frac{1}{2}t^4 - 20t^2 + 66t, \qquad 0 \leq t \leq 4$$

(a) Find:

(i) $\frac{dx}{dt}$;

(3)

(ii) $\frac{d^2x}{dt^2}$.

(2)

(b) Verify that x has a stationary value when $t = 3$, and determine whether this stationary value is a maximum value or a minimum value.

(4)

(c) Find the rate of change of x with respect to t when $t = 1$.

(2)

(d) Determine whether the distance of the car from O is increasing or decreasing at the instant when $t = 2$.

(2)

(Total 13 marks)

2 (a) The polynomial p(x) is given by p(x) = $x^3 + x - 10$.

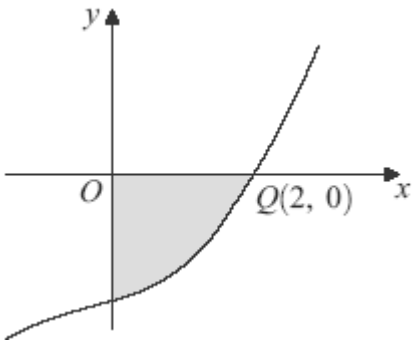
(i) Use the Factor Theorem to show that $x - 2$ is a factor of p(x).

(2)

(ii) Express p(x) in the form $(x - 2)(x^2 + ax + b)$, where a and b are constants.

(2)

(b) The curve C with equation $y = x^3 + x - 10$, sketched below, crosses the x-axis at the point $Q(2, 0)$.



(i) Find the gradient of the curve C at the point Q.

(4)

(ii) Hence find an equation of the tangent to the curve C at the point Q.

(2)

(iii) Find $\int (x^3 + x - 10)\,dx$.

(3)

(iv) Hence find the area of the shaded region bounded by the curve C and the coordinate axes.

(2)

(Total 15 marks)

3 The curve with equation $y = x^5 + 20x^2 - 8$ passes through the point P, where $x = -2$.

(a) Find $\frac{dy}{dx}$.

(3)

(b) Verify that the point P is a stationary point of the curve.

(2)

(c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the point P.

(3)

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point.

(1)

(d) Find an equation of the tangent to the curve at the point where $x = 1$.

(4)
(Total 13 marks)

4 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M.

(a) Find $\frac{dy}{dx}$.

(3)

(b) Hence find the x-coordinate of M.

(3)

(c) (i) Find $\frac{d^2y}{dx^2}$.

(1)

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point.

(2)

(d) Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$.

(2)
(Total 11 marks)

5 (a) The polynomial p(x) is given by p(x) = $x^3 - 7x - 6$.

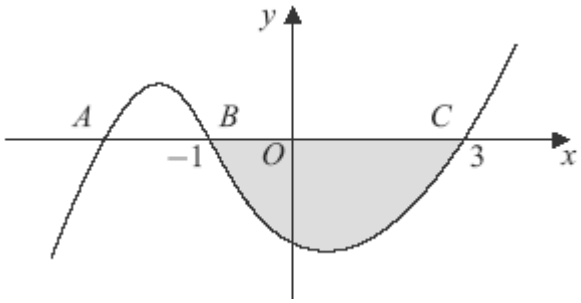
(i) Use the Factor Theorem to show that $x + 1$ is a factor of p(x).

(2)

(ii) Express p(x) = $x^3 - 7x - 6$ as the product of three linear factors.

(3)

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x-axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A.

(1)

(ii) Find $\int_{-1}^{3} (x^3 - 7x - 6)\mathrm{d}x$.

(5)

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C.

(1)

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B.

(3)

(v) Hence find an equation of the normal to the curve at the point B.

(3)

(Total 18 marks)