

Differentiation

Question Paper 5

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 1
Topic	Calculus
Sub Topic	Differentiation
Booklet	Question Paper 5

Time Allowed: 78 minutes

Score: /65

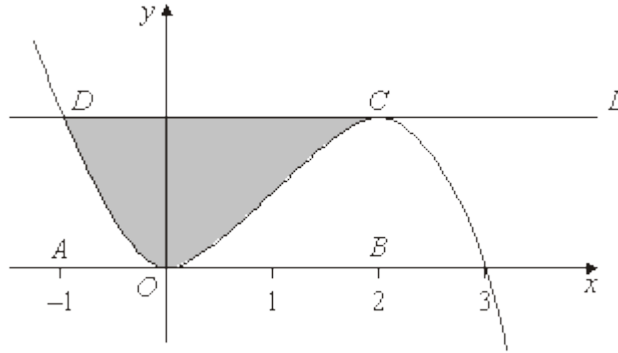
Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1

The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L .



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x -axis at the origin O and crosses the x -axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

- (a) Find the area of the rectangle $ABCD$. (2)
- (b) (i) Find $\int (3x^2 - x^3) dx$. (3)
- (ii) Hence find the area of the shaded region bounded by the curve and the line L . (4)
- (c) For the curve above with equation $y = 3x^2 - x^3$:
- (i) find $\frac{dy}{dx}$; (2)
- (ii) hence find an equation of the tangent at the point on the curve where $x = 1$; (3)
- (iii) show that y is decreasing when $x^2 - 2x > 0$. (2)
- (d) Solve the inequality $x^2 - 2x > 0$. (2)

(Total 18 marks)

2

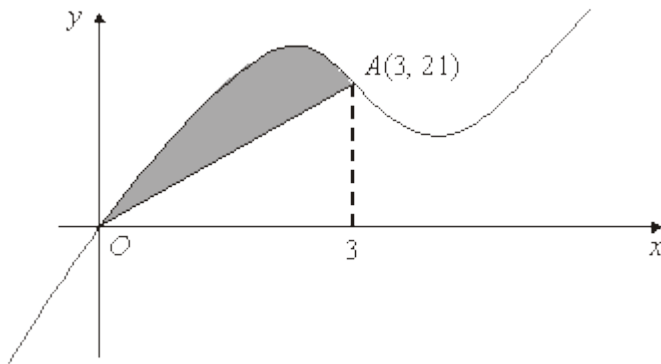
A curve has equation $y = 7 - 2x^5$.

- (a) Find $\frac{dy}{dx}$. (2)
- (b) Find an equation for the tangent to the curve at the point where $x = 1$. (3)
- (c) Determine whether y is increasing or decreasing when $x = -2$. (2)

(Total 7 marks)

3

The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x -axis at the origin O and the point $A(3, 21)$ lies on the curve.

- (a) (i) Find $\frac{dy}{dx}$. (3)
- (ii) Hence show that the curve has a stationary point when $x = 2$ and find the x -coordinate of the other stationary point. (4)

- (b) (i) Find $\int (x^3 - 10x^2 + 28x)dx$. (3)
- (ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)dx = 56\frac{1}{4}$. (2)
- (iii) Hence determine the area of the shaded region bounded by the curve and the line OA . (3)
- (Total 15 marks)**

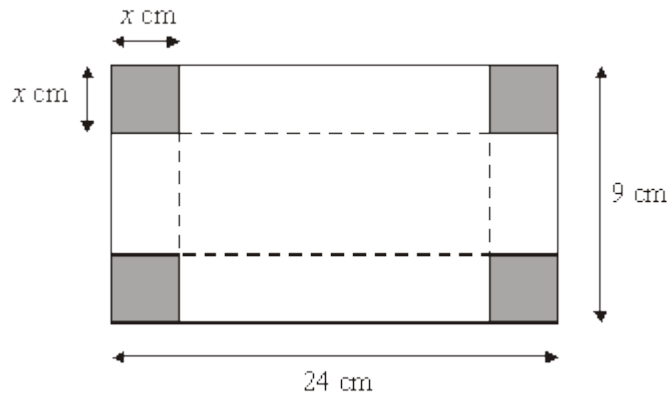
4

A curve has equation $y = x^5 - 6x^3 - 3x + 25$.

- (a) Find $\frac{dy}{dx}$. (3)
- (b) The point P on the curve has coordinates $(2, 3)$.
- (i) Show that the gradient of the curve at P is 5. (2)
- (ii) Hence find an equation of the normal to the curve at P , expressing your answer in the form $ax + by = c$, where a , b and c are integers. (3)
- (c) Determine whether y is increasing or decreasing when $x = 1$. (2)
- (Total 10 marks)**

5

The diagram below shows a rectangular sheet of metal 24 cm by 9 cm.



A square of side x cm is cut from each corner and the metal is then folded along the broken lines to make an open box with a rectangular base and height x cm.

- (a) Show that the volume, V cm³, of liquid the box can hold is given by

$$V = 4x^3 - 66x^2 + 216x$$

(3)

- (b) (i) Find $\frac{dV}{dx}$.

(3)

- (ii) Show that any stationary values of V must occur when $x^2 - 11x + 18 = 0$

(2)

- (iii) Solve the equation $x^2 - 11x + 18 = 0$.

(2)

- (iv) Explain why there is only one value of x for which V is stationary.

(1)

- (c) (i) Find $\frac{d^2V}{dx^2}$.

(2)

- (ii) Hence determine whether the stationary value is a maximum or minimum.

(2)**(Total 15 marks)**