Factor Theorem

Question Paper 3

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 1
Topic	Algebra
Sub Topic	Factor Theorem
Booklet	Question Paper 3

Time Allowed: **75 minutes**

Score: **/62**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The polynomial p(x) is given by

p(x) = $x^3 - 4x^2 - 7x + k$

where k is a constant.

(a) (i) Given that $x + 2$ is a factor of p(x), show that $k = 10$.

(2)

(ii) Express p(x) as the product of three linear factors.

(3)

(b) Use the Remainder Theorem to find the remainder when p(x) is divided by $x - 3$.

(2)

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the x-axis and the y-axis. (You are **not** required to find the coordinates of the stationary points.)

(4)

(Total 11 marks)

2 (a) The polynomial f(x) is given by f(x) = $x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor off(x)

(2)

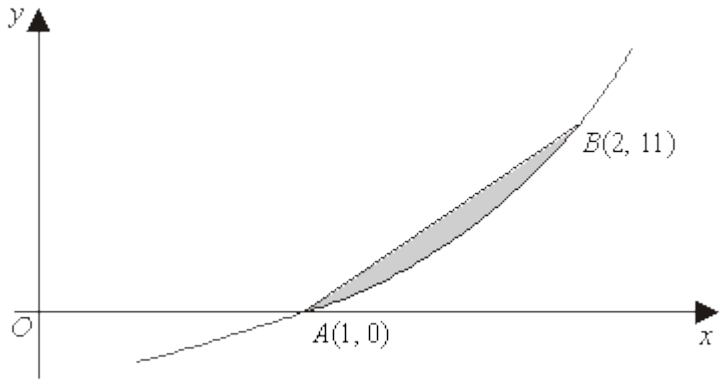
(ii) Express f(x) in the form $(x - 1)(x^2 + px + q)$, where p and q are integers.

(2)

(iii) Hence show that the equation f(x) = 0 has exactly one real root and state its value.

(3)

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point A (1, 0) and the point B (2, 11) lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,dx$.

(3)

(ii) Hence find the area ofthe shaded region bounded by the curve and the line AB.

(4)

(Total 14 marks)

3 The polynomial p(x) is given by

p(x) = $x^3 + x^2 - 10x + 8$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of p(x).

(2)

(ii) Hence express p(x) as the product of three linear factors.

(3)

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.)

(4)

(Total 9 marks)

4 The polynomial p(x) is given by p(x) = $x^3 - 4x^2 + 3x$.

(a) Use the Factor Theorem to show that $x - 3$ is a factor of p(x).

(2)

(b) Express p(x) as the product of three linear factors.

(2)

(c) (i) Use the Remainder Theorem to find the remainder, r, when p(x) is divided by $x - 2$.

(2)

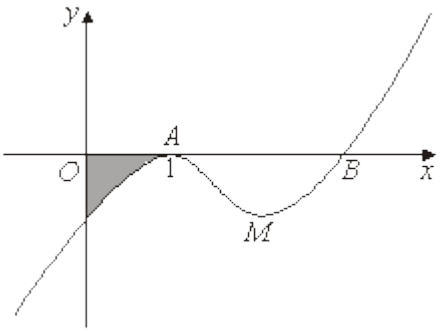
(ii) Using algebraic division, or otherwise, express p(x) in the form

$$(x - 2)(x^2 + ax + b) + r$$

where a, b and r are constants.

(4)
(Total 10 marks)

5 The curve with equation $y = x^3 - 5x^2 + 7x - 3$ is sketched below.



The curve touches the x-axis at the point A (1, 0) and cuts the x-axis at the point B.

(a) (i) Use the factor theorem to show that $x - 3$ is a factor of

$$p(x) = x^3 - 5x^2 + 7x - 3$$

(2)

(ii) Hence find the coordinates of B.

(1)

(b) The point M, shown on the diagram, is a minimum point of the curve with equation $y = x^3 - 5x^2 + 7x - 3$.

(i) Find $\frac{dy}{dx}$.

(2)

(ii) Hence determine the x-coordinate of M.

(3)

(c) Find the value of $\frac{d^2y}{dx^2}$ when $x = 1$.

(2)

(d) (i) Find $\int (x^3 - 5x^2 + 7x - 3)\, dx$.

(4)

(ii) Hence determine the area of the shaded region bounded by the curve and the coordinate axes.

(4)

(Total 18 marks)