

Integration

Question Paper 3

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 2
Topic	Calculus
Sub Topic	Integration
Booklet	Question Paper 3

Time Allowed: 89 minutes

Score: /74

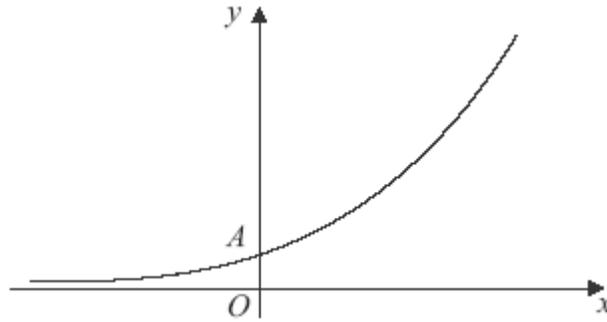
Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1

The diagram shows a sketch of the curve $y = 2^{4x}$.



The curve intersects the y -axis at the point A .

(a) Find the value of the y -coordinate of A . (1)

(b) Use the trapezium rule with six ordinates (five strips) to find an approximate value for $\int_0^1 2^{4x} dx$, giving your answer to two decimal places. (4)

(c) Describe the geometrical transformation that maps the graph of $y = 2^{4x}$ onto the graph of $y = 2^{4x-3}$. (2)

(d) The curve $y = 2^{4x}$ is translated by the vector $\begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$ to give the curve $y = g(x)$.

The curve $y = g(x)$ crosses the x -axis at the point Q . Find the x -coordinate of Q . (4)

(e) (i) Given that

$$\log_a k = 3 \log_a 2 + \log_a 5 - \log_a 4$$

show that $k = 10$. (3)

(ii) The line $y = \frac{5}{4}$ crosses the curve $y = 2^{4x-3}$ at the point P . Show that the x -coordinate of P is $\frac{1}{4 \log_{10} 2}$.

(3)
(Total 17 marks)

2

- (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_{1.5}^6 x^2 \sqrt{x^2 - 1} \, dx$$

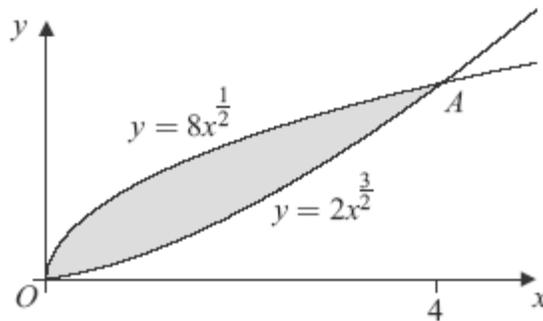
giving your answer to three significant figures.

(4)

- (b) State how you could obtain a better approximation to the value of the integral using the trapezium rule.

(1)**(Total 5 marks)****3**

The diagram shows a sketch of the curves with equations $y = 2x^{\frac{3}{2}}$ and $y = 8x^{\frac{1}{2}}$.



The curves intersect at the origin and at the point A, where $x = 4$.

- (a) (i) For the curve $y = 2x^{\frac{3}{2}}$, find the value of $\frac{dy}{dx}$ when $x = 4$.

(2)

- (ii) Find an equation of the normal to the curve $y = 2x^{\frac{3}{2}}$ at the point A.

(4)

- (b) (i) Find $\int 8x^{\frac{1}{2}} \, dx$.

(2)

- (ii) Find the area of the shaded region bounded by the two curves.

(4)

- (c) Describe a single geometrical transformation that maps the graph of $y = 2x^{\frac{3}{2}}$ onto the graph of $y = 2(x + 3)^{\frac{3}{2}}$.

(2)
(Total 14 marks)

4

- (a) Write down the value of n given that $\frac{1}{x^4} = x^n$.

(1)

- (b) Expand $\left(1 + \frac{3}{x^2}\right)^2$.

(2)

- (c) Hence find $\int \left(1 + \frac{3}{x^2}\right)^2 dx$.

(3)

- (d) Hence find the exact value of $\int_1^3 \left(1 + \frac{3}{x^2}\right)^2 dx$.

(2)

(Total 8 marks)

5

- (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^6 \sqrt{x^3 + 1} dx, \text{ giving your answer to four significant figures.}$$

(4)

- (b) The curve with equation $y = \sqrt{x^3 + 1}$ is stretched parallel to the x -axis with scale factor $\frac{1}{2}$ to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$.

(2)

(Total 6 marks)

6

Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{x^2 + 3} dx$$

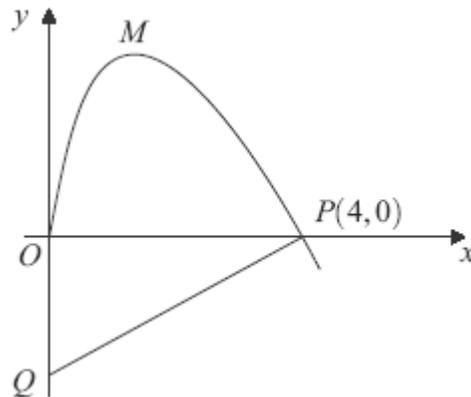
giving your answer to three decimal places.

(Total 4 marks)

7

A curve, drawn from the origin O , crosses the x -axis at the point $P(4, 0)$.

The normal to the curve at P meets the y -axis at the point Q , as shown in the diagram.



The curve, defined for $x \geq 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

- (a) (i) Find $\frac{dy}{dx}$. **(3)**
- (ii) Show that the gradient of the curve at $P(4, 0)$ is -2 . **(2)**
- (iii) Find an equation of the normal to the curve at $P(4, 0)$. **(3)**
- (iv) Find the y -coordinate of Q and hence find the area of triangle OPQ . **(3)**
- (v) The curve has a maximum point M . Find the x -coordinate of M . **(3)**
- (b) (i) Find $\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}} \right) dx$. **(3)**

(ii) Find the total area of the region bounded by the curve and the lines PQ and QO .

(3)

(Total 20 marks)